MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Dr. A. C. D. CROMMELIN, PRESIDENT, in the Chair.

The election by the Council of the following Fellows was duly confirmed :—

Mars F. Baumgardt, D.O.S., Curator of the Clark Observatory, 1208 Hollingsworth Building, 606 South Hill Street, Los Angeles, Cal., U.S.A.;

Dr. Edwin F. Carpenter, Steward Observatory, University of Arizona, Tucson, Arizona, U.S.A.;

Hendon Egerton Houghton, High Commissioner's Office, Cape Town, S. Africa;

Kenelm Jocelyn Townshend Hutchings, Lieut. R.N.R., Arosfa, Deganwy, N. Wales;

Charles Vivian Jackson, Imperial College of Science and Technology, South Kensington, and 70 Addison Road, London, W. 14; and

Miss Violet Florence White, B.Sc., 3 Bonneville Gardens, Clapham Park, London, S.W. 4.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Miss Allie Vibert Douglas, M.B.E., Ph.D., Lecturer in Astrophysics, McGill University, Montreal, P. Quebec, Canada (proposed by W. M. Smart);

James J. Marshall, L.L.D., Secretary, Southern Cross Astronomical Society, Bank of Bay Biscayne Bldg., Miami, Florida, U.S.A. (proposed by C. Ions);

Jan Hendrik Oort, Sterrewacht, Leiden (proposed by F. W. Dyson) ;
John R. Owens, Ffospilcorn, Llanrhystyd, Aberystwyth, Wales (proposed by J. Silas Evans) ; and
Thomas Routledge, B.Sc., School House, Lees Hill, Low Row, Carlisle (proposed by J. Proudman).

Seventy-one presents were announced as having been received since the last ordinary meeting, including :—

Lowell Observatory, Transparency and two Prints of the Trans-Neptunian Planet.
Radcliffe Observatory, Two Transparencies of the Trans-Neptunian Planet.
Swedenborg Society, Collected Works of Emanuel Swedenborg (3 vols.).

The President read the following telegram, which had been received from Lowell Observatory :—

Professor H. H. Turner, Oxford.—Through you, Lowell Observatory wishes to express sincere thanks to the Royal Astronomical Society for its congratulations and hopes the Society will accept photographs Lowell's Planet X that are being forwarded. Slipher.

The President presented the Gold Medal to Dr. J. S. Plaskett.

Dr. Plaskett then delivered the George Darwin Lecture, taking as his subject " The High-temperature Stars."

The High-temperature Stars. (George Darwin Lecture, delivered by Dr. J. S. Plaskett, F.R.S., on 1930 May 9.) (Plate 14.)

Introduction.

It appears to have been the custom at the three George Darwin Lectures hitherto given for the lecturer to choose as his subject one in which he has been personally engaged or to which he has made some contribution of value. It seems that I can hardly do better than follow that precedent, and I have chosen the high-temperature stars as the theme of my remarks this afternoon. The physical properties and the motions of this interesting, but limited, group of stars have been under investigation at Victoria for nearly ten years, beginning in 1920 June with a year's research by H. H. Plaskett on the spectra of three O-type stars. After the completion of an investigation on the absolute dimensions of six eclipsing variables in 1921, I personally commenced the observation of the radial velocities and other spectral characteristics of all

the O absorption, together with some of the emission O-type stars, within reach at Victoria. This was completed in about three years, and was immediately followed, in 1924, by a joint investigation by J. A. Pearce and myself on the radial velocities of all Bo to B5 stars brighter than 7·5 visual magnitude and north of declination − 11°, which had not been previously investigated. The observation and measurement of the 4200 odd spectra of over 600 stars have been completed, but the discussion of the data is still in progress although sufficiently far advanced to obtain some new and interesting conclusions. In addition, C. S. Beals during the past two years has investigated both the ordinary and the red spectral regions of all Wolf-Rayet stars north of − 23° declination. He has just completed the discussion of the results, and a paper on "The Nature of Wolf-Rayet Emission" has recently appeared in *Monthly Notices*. From this review of the work at Victoria on the high-temperature stars it is evident that the results of these investigations should provide material of interest for the George Darwin Lecture.

The high-temperature stars are usually considered as limited to the spectral classes O and B, and may be taken as consisting of the Wolf-Rayet stars and the spectral subdivisions between O5 and B5. The types B8 and B9 show more particularly the characteristics of class A than of class B and are not included in this discussion. The results to be described have been obtained mainly from an analysis of the radial velocities, although naturally these have been supplemented by other published data of this remarkable, but limited, group of stars. Less than one-tenth of 1 per cent. of the stars in the Henry Draper Catalogue are of O type, one-third of 1 per cent. of types Bo to B2, and about 1 per cent. of types B3 and B5. These limited groups are remarkable as including among those observable by the spectrograph the stars of highest temperature, greatest mass, highest luminosity, and greatest distance from the sun in the sky, and their investigation should give results of value in the study of the stellar system. The subject can best be attacked under two divisions, treating, first, the physical properties of the high-temperature stars, and, secondly, their motions.

The Physical Properties of the High-temperature Stars.

The Temperatures.—Our knowledge of the temperatures of the O and B stars is necessarily based mainly on ionization methods, as spectro-photometric measures for such high temperatures, where the maximum energy is in the ultra-violet beyond the observable region, and where a change of temperature from 20,000° to 30,000°, for example, produces only a small change of slope in the observable intensity-distribution curve, are very uncertain. Thus Wilsing's spectro-photometric temperature of a Bo star is 12,300° as compared with Rosenberg's 30,000°, while some recent work by Greaves, Davidson, and Martin, and by Gerasimovič, gives temperatures of the order of 7000° or so for some B-type stars, with other observers giving intermediate values. There is hence considerable uncertainty as to the true colour temperature, as

determined by spectro-photometric methods, of the O- and B-type stars. Although there are also uncertain factors in temperatures determined by ionization methods, such as the estimations of the partial electron pressures and of the positions of maximum strength of the lines, there seems to be ground for reasonable confidence that the deduced temperatures are of the right order. Only a few determinable temperatures corresponding to the maximum strength of the lines of *He*I, *Si*III, *Si*IV, and *He*II are available for the temperature scale of the O and B stars. Individual judgment will play a considerable part in fixing the maxima and in interpolating the temperature to be assigned to intermediate spectral subdivisions. For the purpose of calculating the surface brightness, the difference between visual and bolometric magnitudes, and the temperature correction in the mass-luminosity relation, the following table, which is due to Pearce,* has been used.

Table I.

Temperatures, Surface Brightness, and Magnitude Differences.

Type.	Temp.	Surface Brightness.	$M_v - M_b$.
	° K.	m.	m.
O5	35,000	−4·15	+3·61
O6	33,000	−4·10	+3·41
O7	31,000	−4·03	+3·20
O8	29,000	−3·96	+2·98
O9	27,000	−3·88	+2·76
B0	25,000	−3·78	+2·52
B1	22,000	−3·61	+2·13
B2	19,000	−3·38	+1·73
B3	16,000	−3·08	+1·28
B5	14,000	−2·80	+0·98

Even if these temperatures have to be modified later, the resulting changes in the computation of absolute magnitudes from masses or from assumed densities will be relatively small.

The Masses.—Stellar masses can only be determined directly by means of the orbital motion of binary stars. In the case of visual binaries the dimensions of the orbit are given in arc, and the masses can be obtained only when the parallax is known. The distances of the high-temperature stars are so great that no reliable individual parallaxes are available, so that direct data on the masses of binary stars of types O and B are limited generally to spectroscopic doubles. In spectroscopic orbits the inclination of the orbit plane is unknown unless the star is also an eclipsing variable with a determined light curve. Hence absolute determinations of the masses of binary stars are limited to the cases where the spectroscopic orbits of both components of an eclipsing variable with known photometric elements have

* *Pub. D.A.O.*, **3**, 302, 1926.

been determined. Although there are only six such combined orbits available in the O to B5 stars, there are twenty-one additional double orbits for which the projected masses—the products of the mass of each component into $\sin^3 i$—are known. The average value of $\sin^3 i$ (viz., 0·59) is usually increased, on account of the greater chance of discovery of binaries with high inclination, to 0·67, and consequently the increase of the projected masses by 50 per cent. will give the average masses of these double-lined orbits. Combining these twenty-one average masses with the actual masses of the six combined eclipsing and spectroscopic systems, we get the average mass of the brighter component of five O-type stars as 43 times, of ten Bo to B2 stars as 15 times, and of twelve B3, B5 stars as 6 times the mass of the sun.

There still remain 54 known orbits of O to B5 stars in which only one spectrum is visible, and consequently only the mass function

$$f = \frac{m_2{}^3 \sin^3 i}{(m_1 + m_2)^2} = \mathrm{M}\frac{\alpha^3 \sin^3 i}{(1+\alpha)^3},$$

where α is the ratio of the masses of the two components and $\mathrm{M} = m_1 + m_2$, can be obtained. In order to use this relation statistically, exceptional cases must not be included, and therefore four stars of the type of β Cephei with periods of a small fraction of a day, whose velocity oscillations are probably not due to orbital motion, along with two having orbits derived from the interstellar calcium lines only, were discarded. Similarly the B1 star H.D. 163181, with abnormally large mass function $f = 8{\cdot}84$, which on any reasonable assumption of α and i would have a mass of 200 times the sun's, and eight others with f less than 0·01, whose masses similarly computed would be much less than that of the sun, were also discarded, as being on the one side about 10 times, and on the other less than a tenth, of the average mass of stars of the corresponding type.

The remaining 39 single orbits with the 27 double, 66 in all, were tabulated, the f values of both single and double orbits being calculated where not available, and the corresponding data are given in Table II.,

TABLE II.

Data for 66 O to B5 Orbits.

Type.	Mass Function.				$\frac{\bar{f}_1}{\bar{f}_{11}}$.	$\bar{\alpha}_{11}$	Computed α_{11}
	No.	$\bar{f}_1$	No.	$\bar{f}_{11}$			
O	5	1·62	5	4·91	0·33	0·86	0·47
Bo–B2	10	0·63	10	1·56	0·40	0·75	0·46
B3–B5	24	0·38	12	0·63	0·60	0·67	0·50

where the subscripts refer to orbits with one and two components. The observed mass ratios for the double orbits range between 0·37 and nearly unity, and they must be considerably smaller in the single orbits, for otherwise the second spectrum would be visible. The average observed values of α for the different groups are given in the seventh column, showing a considerable decrease with temperature.

The values of the mass function, f, columns three and five of Table II., are in every group considerably smaller for the single orbits than for those in which the spectra of both components are present. The expression for the mass function $f = M\frac{a^3 \sin^3 i}{(1+\alpha)^3}$ has three variables M, i, and α. As there is no reason to suppose that the total masses or the inclinations of the orbit planes should be different for binaries in which only one spectrum is visible from those which show both spectra, the smaller f values for the single orbits must be due to a smaller ratio of the masses of the two components. As shown by Beer, the average value of α_I can be obtained from the known values of f_I, f_{II}, and α_{II} by the following formula easily obtained from the expression

$$f = M\frac{a^3 \sin^3 i}{(1+\alpha)^3},$$

namely

$$\alpha_I = \frac{\sqrt[3]{\frac{f_I}{f_{II}}} \cdot \frac{\alpha_{II}}{1+\alpha_{II}}}{1 - \sqrt[3]{\frac{f_I}{f_{II}}} \cdot \frac{\alpha_{II}}{1+\alpha_{II}}},$$

and the corresponding values for the three groups are given in the last column of Table II. The masses corresponding to these values of α_I were computed from the f values of the single orbits and were averaged along with those already obtained from the double orbits for each group, and these average values for the masses of the brighter components are given in the fourth column of Table III.

This procedure is somewhat arbitrary when the relatively small numbers of orbits available are considered, and has the disadvantage of making some of the stars with high f terms have abnormal masses. A striking example is the O-type binary 29 Canis Majoris, whose f value is 4·58, giving the brighter component an average mass of 153⊙, considerably larger than the mass assigned to 6° 1309 which has an f value of 13·16, nearly three times as large. This single example explains the larger average mass given to the O stars by this method than by the second, which depends on assumptions of the mass ratio increasing with the magnitude of f.

There is arbitrariness involved here also, but it seems the best that can be done with the limited material and is given for what it is worth. As there are two observed values of α of 0·37 in double orbits, it would appear that α in single orbits may be 0·3 or even lower. On the other hand, if α becomes greater than 0·7 in single orbits, the second spectrum should be seen. The f values of the single orbits in each of the three main divisions of Table II. were arranged into five groups of increasing magnitude of f, and α was assumed as 0·3, 0·4 . . . 0·7 respectively for these groups. This procedure has the advantage of avoiding the abnormally large and small masses given by the formula, and has been preferred for obtaining the masses and luminosities. The mean values of the masses of the brighter components of these binary systems

obtained by this method are given in the fifth column of Table III. and show a more regular progression than by the formula. The average spectral type of the 10 O-type stars is O9, and the temperatures and luminosities in Table III. refer to this type. The higher value of the mass for B5 than for B3 is due to abnormally high masses for three of the seven B5 stars, and is hence of low weight.

TABLE III.

Masses and Luminosities.

Type.	Temp.	No.	Mass by Formula.	Mass, α Assumed.	M_b.	Temp. Term.	M_v-M_b.	M.
	°							
O6–O9·5	27,000	10	47·8	37·8	−5·17	−1·43	+2·76	−3·84
Bo	25,000	6	26·2	24·3	−4·32	−1·36	+2·52	−3·16
B1	22,000	7	14·3	16·9	−3·60	−1·25	+2·13	−2·72
B2	19,000	7	12·8	13·9	−3·20	−1·12	+1·73	−2·59
B3	16,000	29	8·6	7·2	−1·74	−0·98	+1·28	−1·44
B5	14,000	7	10·0	8·0	−1·98	−0·86	+0·98	−1·86
Bo–B2	22,000	20	17·2	18·1	−3·82	−1·25	+2·13	−2·94
B3–B5	15,000	36	8·9	7·4	−1·79	−0·92	+1·13	−1·58

The Luminosities.—The use of Eddington's mass-luminosity relation, which, notwithstanding criticisms of its theoretical basis, well represents the observational data, gives us immediately the luminosities or bolometric absolute magnitudes corresponding to the average masses of the brighter components of spectroscopic binaries, as given in column five, Table III. The absolute visual magnitudes are given in the last column of the table, and the steps by which these are obtained are obvious from the preceding columns. These values of the absolute magnitudes are approximately the same as those given by other methods, as can be seen in Table IV., where various values by parallactic motion, group or cluster parallaxes, and hypothetical parallaxes are collected together.

TABLE IV.

Values of Absolute Magnitude.

Type.	Charlier.	Adams.	Edwards.	Gerasimovič. $m<4·5$.	Gerasimovič. $m>4·5$.	Hypothetical Parallaxes.	Spectroscopic Binaries.	Computed Densities.
O	..	..	..	..	..	−3·8	−3·8	0·049
Bo	−0·9 *	−3·1	−3·4			−3·0	−3·2	
B1	−3·0	−2·5	−2·6	−4·0	−2·1	−3·0	−2·7	0·056
B2	−2·6	−1·8	−1·9			−2·0	−2·6	
B3	−1·2	−1·2	−1·4	−2·1	−0·8	−0·3	−1·4	0·071
B5	−1·0	−0·6	−0·7	−1·3	−0·5	+0·6	−1·9	

* Charlier's low value for Bo is probably due to the inclusion of stars marked B.

To these values should be added my early value of − 4·0 and Wilson's of − 3·7 for the O-type stars. Considering the uncertainty of much of the data on which these luminosities are based, the agreement is remarkably good, and while some of the individual determinations should perhaps be looked on with doubt, the agreement among all is relatively so close as to give confidence in the final mean result. A further indirect confirmation of the substantial correctness of the masses derived from spectroscopic binaries and the luminosities derived from the mass-luminosity relation may be obtained by computing the densities from the surface brightnesses of Table I. and the masses and luminosities of Table III. The resulting densities are given in the last column of Table IV. and agree well with the mean of the densities obtained from the light curves of the six eclipsing systems among the binaries used, this mean density being 0·070.

While this method of obtaining the average masses of the O to B5 stars requires the use of rather uncertain assumptions as to the mass ratio, the same criticism is true of all methods of obtaining parallaxes or luminosities from these very few and very distant objects. It does provide an independent method of determining the masses and luminosities of the high-temperature stars, and the agreement of the derived luminosities by the several greatly differing methods gives additional confidence in the substantial correctness of the results.

The Luminosities of the Wolf-Rayet Stars.—The question of the absolute magnitudes of the Wolf-Rayet stars is an important one owing to their unique character and position in the spectral sequence. Unfortunately, not much information is available about the mean parallax or absolute magnitudes of these objects. Wilson's * results do not clearly separate the Wolf-Rayet stars from the absorbing O-type with emission lines, but, so far as can be judged, the Wolf-Rayet stars are about 0·7 or 0·8 magnitude fainter than the absorbing O-type stars. Some additional evidence is provided by the small group of O- and B-type stars around P Cygni, which are generally believed to form a single compact cluster in which differences in apparent magnitude will correspond to differences in absolute magnitude. The average apparent magnitude of six Wolf-Rayet stars is 0·8 magnitude fainter than that of five absorbing O-type stars. This difference agrees with Wilson's and adds to confidence in the results. At my request, Dr. Beals, who has recently completed a valuable discussion on the Wolf-Rayet stars in which a useful hypothesis on the nature of the Wolf-Rayet emission has been developed, obtained a spectrum of a tenth magnitude Wolf-Rayet star near the h and χ clusters in Perseus. Although no stellar absorption lines were present to obtain the velocity of the star, the "interstellar" H and K lines gave a radial velocity, after correction for solar motion, of − 34 km. per sec. Applying the rotational effect to this velocity, we find it corresponds to a distance of the centre of gravity of the interstellar matter of about 2000 parsecs. From our recent results in *Monthly Notices* the star will be twice that distance, or 4000 parsecs, corresponding to an absolute magnitude

* *A.J.*, **38**, 1, 1927.

− 3·0. It is unfortunate that this same method could not be applied to the six Wolf-Rayet stars in the P Cygni group as the "interstellar" velocities are well determined, but these stars are in a position in galactic longitude perpendicular to the direction to the centre, where the rotational effect is practically zero. Both the other methods, assuming the absolute magnitude of stars of type O to be − 3·8, as above obtained, give practically the same result, and, while such a close agreement is probably accidental, the intrinsic luminosity of these puzzling objects appears to be about 0·8 magnitude fainter than that of the absorption O-type stars. The great difference of 6 or 8 magnitudes between the absolute magnitudes of the Wolf-Rayet stars and the nuclei of planetary nebulæ is, however, no nearer explanation by these results.

These values of the masses and luminosities of the high-temperature stars are in good agreement with all known data about these very distant objects and are probably approximately correct for those that have been observed. There is now, however, some evidence pointing to the presence of O- and B-type stars 2 or 3 magnitudes absolutely fainter than those obtained above. This appears from Wilson's * and my † results on the O-type stars, where the apparently faint stars are of smaller absolute magnitude than the bright, and is also evident from Gerasimovič's results for the B-type stars as given in Table IV. This difference can be explained as due to larger errors in the proper motions of faint stars (producing spuriously large proper motions and parallaxes), but some recent work by Pannekoek ‡ on the distribution of the B-type stars seems to confirm a larger dispersion than was previously supposed. His values depend upon the assumption that the B-type stars, which appear to cluster in position in the sky, have nearly the same depth, and hence the absolute magnitudes vary approximately with the apparent. Consequently, although the results obtained in this section probably represent the masses and luminosities of the apparently and absolutely bright O- and B-type stars, the possibility of stars of the same classes some 2 magnitudes fainter, intrinsically, should be kept in mind.

The Motions of the High-temperature Stars.

Observational Data.—The numerical data in regard to the Victoria observations have been given in the introduction and need not be repeated. It may be added that the number of O to B5 stars for which velocities were obtained only at Victoria is about 500, most of which are fainter than visual magnitude 5·5. The 500 brighter stars of these classes have been observed at many institutions, although the Lick and Yerkes Observatories are responsible for the majority. It was a considerable task to collect all these observations, assign suitable weights, and obtain mean values and probable errors. The general catalogue

* *A.J.*, **38**, 1, 1927.
† *Pub. D.A.O.*, **2**, 325, 1924.
‡ *Pub. Ast. Inst. Amsterdam*, **2**, 31, 1929.

thus compiled contains the radial velocities of 995 O to B5 stars. The difficulty and uncertainty of measurement of O- and B-type spectra have been frequently referred to, but I do not wholly agree about such a difficulty. Over 40 per cent. of the 680 stars finally used in the discussions have radial velocities with a probable error of only 1 km. per sec. or less, while, except in a few special cases, no velocities used had a probable error greater than 3 km. per sec. Many of the stars in the catalogue could hardly be said to be insufficiently observed, as for σ Scorpii the radial velocities from 774 plates at four observatories, and for β Cephei from 755 plates at six observatories, have been published. Two other stars, 12 Lacertæ and ϕ Persei, with 368 and 350 spectra respectively, have also been sufficiently observed. Generally speaking, Victoria has not been guilty of contributing to these large numbers, although temporary summer assistants have obtained spectra of β Cephei and 12 Lacertæ for special studies of these short-period variables. Although useful information has been obtained about these four stars, the energy devoted has been quite disproportionate to the results obtained, and would have been much more usefully employed if the 2300 spectra obtained had been distributed among 400 instead of 4 stars.

The 500 brighter stars observed elsewhere than at Victoria required about 1000 entries in the catalogue, indicating that each star was observed on the average at two institutions. This was by no means waste effort, as the average difference in velocity of the same star by different observatories is about 5 km. per sec. The systematic differences, however, are satisfactorily small, generally less than 1 km. per sec., and in view of the large accidental differences, no attempt was made to apply systematic corrections to O- and B-type velocities. In the preparation of the catalogue care was taken to differentiate as far as possible the stars of variable velocity—spectroscopic binaries—from stars of constant velocity. In this process many previously announced B-type binaries were assigned, from later knowledge and experience, to the constant-velocity class. Altogether there are 315 out of 995 stars —nearly one-third—which are probably of binary character, although there are many cases on the border line, which additional observations may transfer from the one side to the other.

Certain of these stars, on account of uncertainty in velocity, incorrect spectral classification, or abnormally great velocity, should not be included in any statistical discussion. Although the exclusion of the 315 binaries might be thought essential, it must be remembered that 80 binaries have their orbital elements, and hence the velocity of the system, determined; while many others with a small range or large number of observations have reliable mean velocities. The criterion for rejection, except in a few special cases, was a probable error greater than $\pm$ 3 km. per sec., and 241 stars were excluded for uncertain velocity. It was found that 49 stars in the list were incorrectly classified, being actually of type B8 or later, and were consequently excluded. Further, 25 stars with a residual velocity—the velocity after removal of the solar component—greater generally than 4 times the random motion or about 40 km. per

sec., were also excluded. There remained some 680 O to B5 stars suitable for the analytical discussion of the motions. The distribution of these 680 stars is shown on the chart (Plate 14) in which the concentration to the galaxy of the fainter stars and Gould's inclined belt of the brighter stars are apparent.

Analysis of Motions: Introduction.—The motions of the O- and B-type stars have long been recognized as showing some peculiar characteristics. As long ago as 1903 Frost and Adams, from the radial velocities of 20 B-type stars, were the first to point out a fact which Campbell later confirmed from much more extensive observational material, that the residual radial velocities of the B-type stars—the velocities after the removal of the solar component—were much smaller than those for any other spectral class; and, further, that there was a large excess of positive velocities—on the average of nearly 5 km. per sec.—as if the system of B stars was expanding with respect to the sun. Various attempts to explain this K term, as Campbell called it, as due to errors in wave-length, personal equation in measurement of the broad lines, asymmetry in contour due to some physical cause, or the Einstein redward shift of the spectral lines of massive stars, could not be made to account for more than part of the 5 km. per sec. observed. A recent attempt by von Pahlen and Freundlich, by the assumption of a special kind of highly eccentric orbital motion, is rendered improbable by the practical disappearance of the K term from the motions of the fainter B-type stars.

The early analyses of the motions of the B-type stars, mainly by Campbell, were performed for the purpose principally of determining the solar motion for this particular group, while the K term appeared as an unexplained positive residual velocity. The possibility of systematic effects other than the solar motion and star-streaming—very small, however, in B-type stars—in the radial velocities was not realised until Lindblad advanced the theory of a rotation of the galaxy. Oort,* in a fine series of papers, developed the mathematical consequences of such a rotation, and showed that among other distant stars those of types O and B distinctly showed the presence of a systematic term most simply explained as due to a differential rotation of the galaxy. It is obvious, if there is such a systematic effect in the radial motions of these stars, this or any other systematic motion must be taken into account in addition to the solar motion before we can speak intelligently of a K term or of the true random or peculiar motions. The appearance of Oort's first paper in 1927, before the radial velocities of the B-type programme were completed, was a temptation to test the rotational hypothesis from the available velocities of these fainter and hence probably more distant stars. The results of this analysis were given in *Monthly Notices* † and strongly confirm the presence of a rotational term in the O- and B-type velocities.

General Treatment.—It seems necessary to give some explanation of the plan followed in discussing the motions of these stars, as some of

* *B.A.N.*, **120**, 132, 159, 1927.
† *M.N.*, **88**, 395, 1928.

the results are necessarily provisional. The Victoria radial velocities were completed nearly a year ago, and several preliminary solutions for the galactic rotation and the K term were made from provisional radial velocities of 875 stars. No attempt was made to determine the solar motion from these velocities, as should be done for any final results, but the component of the usual solar motion was removed and the remaining residual velocities solved for rotation and the K term. These are discussed in the following two sections, but it should be remembered that these solutions are preliminary only, and when the solar motion appropriate to the high-temperature stars instead of that from the naked-eye stars is used there may be some changes in the details of the rotational and K terms, though not, I believe, in their essential features.

The critical discussion of the radial velocities of 995 stars with the exclusion of uncertain velocities, etc., described in a preceding section, was only completed about three months ago, so that there has not been time to make all the detailed solutions required for the complete analysis of the 680 reliable velocities. However, the results of solutions of four main-type and magnitude groups, and also a combined solution of all the stars for the solar motion, the galactic rotation, and the K term, are given in a subsequent section on the solar motion. As the velocity is over 2 km. per sec. greater than the normal and the apex is displaced over 15°, it will be desirable to repeat the detailed solutions for the galactic rotation and the K term, but this will have to appear in a future paper.

Solutions for Galactic Rotation.—After the completion of the radial velocities some months ago, and before the critical discussion of all O- and B-type velocities just described and the preparation of the general catalogue of 995 stars, some preliminary solutions for the galactic rotation were made from the radial velocities of 875 stars, using the methods and equations of Oort. In the general form, assuming the usual solar motion, the rotational expression as applied to radial velocities takes the form

$$\overline{\mathrm{V}} = \mathrm{V}_0 \cos \overline{\mathrm{D}} + \bar{r}\mathrm{A} \sin 2\,(\overline{l} - l_0) \cos^2 \overline{b} + \mathrm{K},$$

where V, V_0, D, and K have their usual meanings, l and b are galactic co-ordinates, l_0 the longitude of the gravitational centre, r the distance of the star from the sun, and A the rotational term (the change in velocity produced by a galactic rotation for stars at a distance of one parsec). The average of the values of A, as determined by Oort and myself, is 0·017 km. per sec. per parsec, or a rotational effect of 1 km. per sec. will be given for stars 60 parsecs, about 200 light-years, from the sun. The barred terms are used, as owing to the disturbing effects of random motions it is advisable to use small groups over a limited range, 20° or 25°, in longitude rather than individual stars. In these preliminary solutions the usual solar motion of 20 km. per sec. towards $\alpha = 271°$, $\delta = +28°$, was assumed, and its component removed from the radial velocity, leaving the residual radial velocity ρ. The

$\cos^2 b$ term may be neglected for stars within 20° of the galaxy, and the above formula is then simplified to

$$\bar{\rho} = K + \bar{r}A \sin 2(\bar{l} - l_0).$$

While preliminary solutions of a number of groupings were made showing the undoubted presence of a rotational effect, the most convincing demonstration of the objective reality of the rotation of the galaxy is provided by arranging the most distant of these high-temperature stars into groups, according to relative distance, by the method used in the recent investigation of the motions of the interstellar matter in *Monthly Notices*, **90**, 243, 1930. The uniform distribution of the interstellar matter throughout the part of the stellar system inhabited by the O- and B-type stars, so clearly demonstrated in that paper, carries as a consequence the possibility of arranging the high-temperature stars, more accurately than by any other method, into groups according to relative distance by means of the intensity of the "interstellar" H and K lines. The solutions from the equation just given of three such groups of the most distant O- and B-type stars and of the intervening interstellar matter gave values of the rotational term $\bar{r}A$ from 6 to 12 times the probable error. While this is a definite indication of the objective reality of the rotational effect, a much more convincing test is given by a comparison of the observed residual velocities of stars and intervening matter with the velocities computed from the values given by the solutions. Such a comparison is given in Table V. for three groups of high-temperature stars which, from the values of $\bar{r}A$ of + 10·2, + 14·5, and + 27·5, and the value of A given above of 1 km. per sec. for stars 200 light-years distant, are at average distances of 2000, 2900, and 5500 light-years from the sun.

TABLE V.

Comparison of Velocities.

Intensity of Interstellar Lines 4·4 to 6·9.

Group.	No.	Mean Long.	$\bar{r}A$ = + 10·22. Stellar Velocities. Observed.	Computed.	$\bar{r}A$ = + 5·03. Cloud Velocities. Observed.	Computed.
		°				
1	2	345	+36·2	+10·8	−1·2	+3·3
2	2	17	+ 5·9	+14·0	+4·9	+4·9
3	14	38	+10·4	+ 9·9	+5·7	+2·9
4	11	61	+ 6·4	+ 2·1	−0·4	−1·0
5	13	75	− 7·3	− 2·4	−0·4	−3·2
6	8	89	− 6·8	− 5·3	−6·0	−4·6
7	8	113	− 2·1	− 5·0	−6·9	−4·4
8	6	131	+ 4·5	− 0·5	+2·7	−2·2
9	8	156	+ 7·0	+ 8·0	−1·7	+2·0
10	13	174	+14·4	+12·6	+4·9	+4·1
11	3	205	− 2·6	+ 9·8	+3·4	+4·2

Table V.—*continued.*

Intensity of Interstellar Lines 7·0 to 7·9.

Group.	No.	Mean Long.	$\bar{r}A = +14{\cdot}53$. Stellar Velocities. Observed.	Computed.	$\bar{r}A = +6{\cdot}91$. Cloud Velocities. Observed.	Computed.
		°				
1	8	12	+ 5·5	+14·3	+ 4·6	+ 6·7
2	12	38	+ 8·9	+ 8·0	+ 6·3	+ 3·7
3	11	51	+ 7·2	+ 1·7	+ 4·8	+ 0·6
4	10	65	− 1·5	− 5·3	− 0·9	− 2·6
5	4	75	− 7·8	− 9·5	− 2·4	− 4·7
6	8	89	−20·2	−13·7	−10·6	− 6·7
7	3	110	−23·9	−13·9	− 9·7	− 6·7
8	3	139	+ 1·7	− 3·2	− 0·3	− 1·7
9	9	160	+10·6	+ 7·2	+ 0·2	+ 3·3
10	10	174	+17·2	+12·2	+ 2·7	+ 5·6
11	1	192	+44·6	+14·4	+13·0	+ 6·7

Intensity of Interstellar Lines 8·0 to 9·5.

			$\bar{r}A = +27{\cdot}52$.		$\bar{r}A = +13{\cdot}72$.	
		°				
1	2	358	+35·7	+24·3	+ 8·2	+ 9·4
2	2	17	+12·2	+29·3	+ 6·1	+11·9
3	6	34	+21·5	+21·3	+ 7·5	+ 7·9
4	2	47	+11·8	+10·4	+ 6·6	+ 2·4
5	12	70	− 3·4	−10·4	− 5·4	− 8·0
6	4	97	−26·8	−23·4	−15·5	−14·3
7	9	107	−28·4	−23·5	−17·7	−14·5
8	4	141	− 4·1	− 0·9	+ 1·2	− 3·2
9	2	169	+13·6	+16·0	+ 1·9	+ 5·3

A mere inspection of the run of the observed residual velocities with those computed from the above values of $\bar{r}A$ for the three groups of stars, especially when the possible disturbing effects of large random motions in groups with few stars or of accidental errors is considered, furnishes the most convincing evidence of the actual presence of the differential motions produced by a rotation of the galaxy. This evidence is made even stronger by the presence of similar differential motions in the intervening diffuse matter, in which, indeed, it almost appears as if the individual molecules obey the rotational swing. This is clearly shown by the close correspondence of the observed and computed velocities for the interstellar matter in each of the three groups. Further, the value of $\bar{r}A$ for this matter is in each group almost exactly half that for the corresponding stars; in other words, the centre of gravity of each group of interstellar matter is half-way between the corresponding group of stars and the sun, or the matter must be uniformly distributed throughout the stellar system. The distribution of the residual velocities of both stars and interstellar matter is so nearly identical with that which would be given by a differential

rotation of the galaxy as to leave no reasonable doubt that these distant stars and the intervening matter are in rotation about a very distant and massive centre.

Solutions for the K Term.—The undoubted presence of this systematic rotational effect in the residual motions of the O- and B-type stars makes the earlier solutions of Campbell and others, who had no knowledge of such systematic motion, of doubtful value so far as the K term or the random or peculiar motions of these stars are concerned. For the preliminary solutions for the K term, the 875 stars were first divided into two temperature or spectral class groups of 248 O to B2 and 627 B3, B5 stars. Further, as the stars used in the early investigations were all brighter than the 6th magnitude, these two type groups were each divided into two magnitude groups, brighter and fainter than 6·0 vis. mag., the arrangement being shown in the first part of Table VI. After division into suitable sub-groups in longitude these four groups were solved for $\bar{r}$A, l_0, and K, with results as given in the upper part of Table VI. The galactic rotation is again confirmed, the rotational term $\bar{r}$A being from 5 to 8 times the probable error except for the B3, B5 stars brighter than the 6th magnitude, where the irregular random motions are of the same order of magnitude as the rotational effect, thus making the values of $\bar{r}$A and l_0 indeterminate. The mean value of the direction to the gravitational centre, 326°·4, agrees almost exactly with Shapley's latest value—327°—of the direction to the centre of the system of globular clusters—thus furnishing additional confirmation of the reality of the rotational motions.

The K terms in these solutions practically vanish, at least well within the probable errors, for all stars fainter than the 6th magnitude. This would seem to indicate that, in general, this mysterious K term cannot be wholly due to errors of wave-length or measurement, or to the Einstein redward shift, as these causes should operate generally for faint as well as for bright stars. It must be remembered, however, that the fainter stars were observed at Victoria, where the most recent laboratory wave-lengths were used, and the brighter stars at various observatories, principally Lick and Yerkes, generally at much earlier dates, with different systems of adjusted wave-lengths, so that some small systematic difference due to wave-length would be possible. Again, part of the difference in the K term for bright and faint stars might be due to a greater Einstein displacement for the apparently bright stars which, as indicated in the first section, might have a greater intrinsic brightness, and hence greater mass and Einstein shift. In an attempt to trace further the origin of this K term—this apparent expansion of the system of B stars—it was felt desirable to make further divisions into type and magnitude groups.

Consequently, the 875 stars were arranged into three type groups of 63 O type, 185 Bo to B2, and 627 B3, B5 stars, and these were subdivided into magnitude groups as shown in the lower part of Table VI. The dividing magnitude 5·5 may be taken as differentiating generally those observed at Victoria from those obtained elsewhere. The solutions in these cases were made for $\bar{r}$A and K only, l_0 being assumed as 325°.

TABLE VI.

Preliminary Solutions for the K *Term.*

Type.	No.	Mag. Interval.	m.	$\bar{r}$A.	K.	l_0.
						° °
O–B2	132	6·0	4·40	+ 4·5±0·9	+6·2±0·7	337·9±10·9
O–B2	116	6·0	6·89	+15·4 1·9	+0·9 1·4	323·9 6·9
B3, B5	355	6·0	4·85	+ 0·7 1·1	+4·6 0·7	Indeterminate.
B3, B5	272	6·0	6·78	+ 3·8 0·7	0·0 0·5	321·9±10·1
			$\bar{m}$			ρ.
O	14	5·5	4·55	+ 4·0 2·3	+6·4 1·6	12·2
O	17	5·5–6·5	5·95	+16·4 3·5	+7·2 2·5	16·5
O	32	6·5	7·30	+21·2 2·7	+4·0 1·7	16·8
Bo–B2	54	4·5	3·18	− 0·7 1·4	+5·8 0·9	9·0
Bo–B2	39	4·5–5·5	5·03	+ 9·2 1·9	+4·8 1·1	10·5
Bo–B2	39	5·5–6·5	6·04	+16·3 1·8	+0·2 1·2	11·4
Bo–B2	53	6·5	7·15	+13·2 2·0	−0·5 1·4	12·1
B3, B5	294	5·5	4·65	− 0·4 0·9	+4·8 0·6	8·0
B3, B5	157	5·5–6·5	6·07	+ 2·8 1·2	+1·4 0·8	8·7
B3, B5	176	6·5	7·01	+ 3·6 0·8	−0·5 0·6	10·3

The behaviour of the K term is interesting and suggestive. In the first place, although it practically vanishes for the B-type stars fainter than 5·5 magnitude—in other words, for the stars obtained at Victoria and hence only in the Northern Milky Way—it remains at about its original value for the brighter B-type stars, for those determined at other observatories and distributed generally all around the galaxy. Further, it persists without appreciable decrease for both bright and faint O-type stars, practically all obtained at Victoria. Obviously one single simple explanation will not suffice for both O- and B-type stars, and further analysis seems necessary.

Group Motions of Southern Stars.—Considering in the first place only the B-type stars, some information may be obtained from a comparison of the mean observed residual velocities for the various longitude groups in each type and magnitude division with those computed from the values of $\bar{r}$A and K obtained from the solutions and given in Table VI. It appears that the major part of this high positive residual velocity, this K term, arises in the Southern Milky Way. Thus in the Bo to B2 stars brighter than 5·5 magnitude the average residual velocity for 41 stars between longitudes 335° and 170° in the northern sky is + 2·6 km., while for 62 stars between 170° and 335° in the southern sky it is + 6·6 km. per sec. For the B3, B5 stars brighter than 5·5 magnitude the average residual velocity for 121 northern stars between 340° and 170° is + 1·6 km., while for 176 southern stars between 170° and 340° it is + 7·0 km. per sec. It is obvious, therefore, that there is a rather large systematic difference between the residual velocities of the northern and southern

B-type stars. These comparisons are shown in detail in a recent paper,* and the conclusion was there reached that the high K term for the brighter B-type stars was mainly due, after the removal of galactic rotation, to a high systematic velocity of recession—probably group or cluster motions of the southern high-temperature stars—most marked between galactic longitudes 240° and 290°. Although Kapteyn has shown a marked community of motion in the southern high-temperature stars, this was obtained by first removing a K term of + 4·3 km. per sec., leaving a residual systematic motion of 18·3 km. per sec., with a vertex at $\alpha = 274°{\cdot}5$, $\delta = +42°$, differing little from the reflex of the solar motion. Indeed, as will be seen below, the solar apex determined from the radial velocities of all the O- and B-type stars is very close to Kapteyn's vertex. While it seems likely that solutions using the solar motion proper to these stars will reduce the differences and make the K term smaller, there will undoubtedly remain an excess of velocities of recession in the southern sky—a K term—requiring the assumption of group motions to explain its presence.

Effect of the Einstein Red Displacement.—Even the above explanation does not help the case of the O-type stars where the K term is present in both northern and southern stars and shows little indication of decrease as the stars become fainter. While it had been earlier suggested that this K term might be due to the slowing of the atomic clock in massive stars—the Einstein redward displacement—it was generally believed that this could have an order of only about one km./sec. or less and was quite insufficient to account for the K term. I am indebted to the insistence of my son, H. H. Plaskett, that this Einstein shift might be a vital factor in the K term, for the decision to make a new calculation of the magnitude of this displacement. The estimates of the masses and luminosities, with the resulting calculated densities of the high-temperature stars, obtained in the first section provide the most reliable data hitherto available. Taking the masses in the fifth column of Table III. and the calculated densities in the last column of Table IV. we get the redward displacement for an O9 star as + 2·5 km. per sec., of a B0 to B2 star + 1·6, and of a B3, B5 star as + 1·0 km. per sec. These values for the B-type stars materially reduce the magnitude of the K term for the brighter stars and practically entirely remove the positive residual in the brighter northern stars, leaving only the K term for the southern stars to be explained by group motion. While in the O-type stars a shift of + 2·5 km. per sec. was obtained for an O9 star of density 0·05, the displacement would probably be greater for the earlier types. The average densities given by eclipsing binaries of O and B type are considerably greater than those calculated in Table IV., and there is the further possibility that the density of single stars might be somewhat greater than that of the components of binaries. The density of Y Cygni, an O9·5 star, is nearly 0·2, and with this density and a mass of 50 times the sun, neither of these assumptions being unreasonable, the redward shift, + 5·0 km. per sec., is ample to explain the K term in O-type stars.

* *Science*, **71**, 225, 1930.

While some of these results, especially those dealing with the group motions of the southern stars, may be modified when the solar motion appropriate to these high-temperature stars is used instead of the usual value, it seems safe to state that this mysterious positive residual, this K term, arises partly from the Einstein redward displacement of the spectral lines of these very massive stars and partly from systematic group motions of recession in the brighter southern B-type stars. However, the ultimate solution of the origin of the K term will apparently have to wait until the establishment of a large observatory in the southern sky with suitable spectroscopic equipment enables the radial velocities of the fainter southern high-temperature stars to be obtained. In no field of astronomical research can such important results be obtained with such a relatively insignificant amount of observational data as in the completion of this analysis over the whole sky.

Peculiar or Random Motions.—The actual peculiar or random velocities of the high-temperature stars after the removal of the solar motion, the galactic rotation, and the K term are given in the last column of the second part of Table VI. They are somewhat higher than those previously given by Campbell, and show a well-marked tendency to increase not only with temperature, but also with faintness or distance. Exactly what this means is not clear, although it must be remembered that in all these solutions the usual solar motion was assumed. Whether this effect will persist after the solar motion, as derived from these stars and given below, is applied cannot be determined until new solutions are made.

Solutions for the Solar Motion.—The previous results have dealt with solutions partly of a preliminary character, and mainly for the purpose of testing the rotation of the galaxy and investigating the nature of the K term. They included many stars whose velocities were somewhat uncertain, and after the completion of the general catalogue about three months ago and the rigorous exclusion of uncertain velocities, etc., it was felt desirable to carry through new solutions from the 680 remaining O- and B-type stars, for the solar motion, the galactic rotation, and the K term simultaneously. The equation used includes the three usual terms required to define the solar motion, the two for the galactic rotation, and the K term, and is as follows :—

$$\overline{V} = X \cos L \cos B + Y \sin L \cos B + Z \sin B + \bar{r} A \sin 2(\overline{l} - l_0) \cos^2 \overline{b} + K,$$

where X, Y, Z are the three components of V_0, and L and B the galactic co-ordinates of the apex. The experience gained in the previous solutions was useful in determining the most suitable way of dividing the stars into type and magnitude groups. As there is a marked break in the rotational effect between B2 and B3, the stars were arranged into the two groups of O to B2, and B3, B5. Further, as there also seems a similar discontinuity at magnitude 5·5, the stars brighter than 5·5 having small and practically indeterminate values of $\bar{r}A$, while for the fainter stars there appears to be little change with increasing magnitude, two magnitude groups were formed, thus making four main groups for the solutions.

It was realized that these stars are unsuitably distributed for determining the solar motion, and particularly unsuited, when they are collected so close to the galactic plane, for the determination of the latitude of the apex. Owing to the large variation previously found in the magnitude of the rotational term for different groupings, it was felt necessary to solve the four groups separately, even though the number of stars was so small and the number of unknowns so large that the individual solutions would have low weight. The solutions of the four groups, all of which have been made by Mr. Pearce, have been carried through in three ways—(*a*) for all six unknown elements, (*b*) for five unknowns, considering the K term zero, (*c*) for four unknowns, fixing the longitude and latitude of the apex at the usual positions 21°·8 and 20°·0. The values are given in Table VII. and are of considerable interest. Owing to lack of time the probable errors for the separate solutions have not yet been computed. They would probably be high, although for the combined solution, using all the stars, they are satisfactory. The latitude of the apex for the O to B2 stars fainter than 5·5 was indeterminate on account of the close concentration to the galaxy, coming out as − 47°, an impossible position. Consequently the solution was repeated with the latitude assumed as 21°·5, the value obtained in a general solution for the solar motion and K term only.

TABLE VII.

General Solutions for Solar Motion.

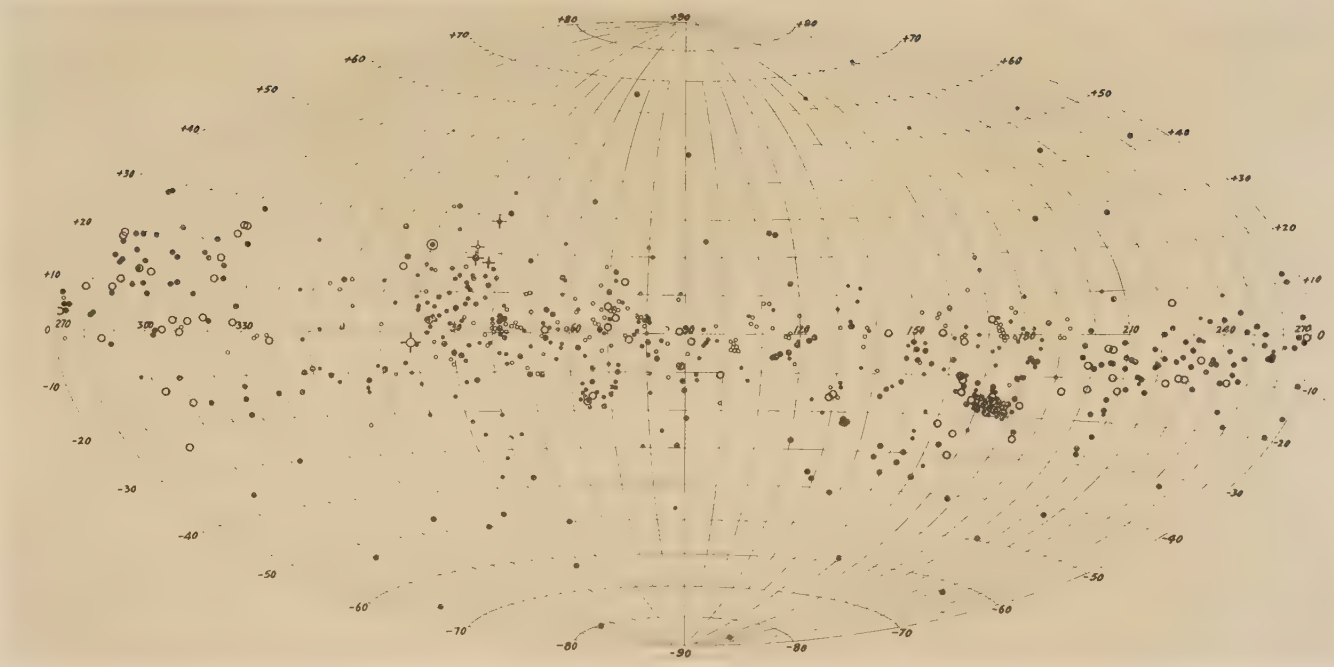
	O–B2.			B3.	B5.	All.
Terms	<5·5	>5·5	>5·5	<5·5	>5·5	
No.	78	139	139	241	222	680
m	3·98	6·60	6·60	4·64	6·60	
K	+ 5·0	+ 2·4	+ 2·1	+ 3·8	+ 2·8	+ 3·5±0·5
V_0	19·1	28·1	21·4	22·8	24·3	22·0±1·0
L	18°·7	31°·6	33°·2	33°·0	36°·7	36°·8±2°·1
B	− 2°·8	−46°·8	(+21°·5)	+19°·2	+28°·0	+17°·3±6°·1
$\bar{r}$A	+ 2·8	+15·8	+17·3	− 1·0	+ 3·2	+ 4·3±0·8
l_0	289°·9	320°·5	321°·2	299°·7	346°·3	321°·5±1°·9
α	291°	334°	274°	276°	267°	281°
δ	+16°	− 2°	+38°	+38°	+43°	+40°
V_0	22·3	26·1	..	25·1	23·6	22·6
L	22°·1	22°·4	..	34°·6	27°·0	33°·2
B	+18°·0	−48°·0	..	+27°·0	+29°·3	+24°·5
$\bar{r}$A	+ 3·8	+17·3	..	+ 0·2	+ 5·3	+ 4·8
l_0	272°·9	319°·4	..	286°·1	342°·3	326°·3
K	+ 4·3	+ 0·1	..	+ 3·9	+ 0·1	
V_0	19·8	17·4	..	22·9	22·3	
*r*A	+ 2·5	+17·0	..	− 0·8	+ 5·5	
l_0	287°·9	319°·7	..	338°·6	341°·2	

The solutions are very interesting, though time does not permit any but the briefest discussion. In both groups of the O to B2 stars the solar apex, particularly the latitude, is poorly determined, but there seems to be no doubt from the more numerous and better distributed B3, B5 stars and from the combined solution that the longitude of the apex is about 15° greater than the usual value of 21°·8. The six positions of the apex in the table given in galactic co-ordinates are translated into equatorial co-ordinates and given immediately below the first section of the table, the last, from the solution for all the stars, being accepted as the best value. These positions are shown in the chart (Plate 14) by the star-pointed open and filled circles, the symbols for the apices being similar to those for the corresponding groups of stars. The apex for the O- and B-type stars is therefore considerably north and east of the usual position, and the solar velocity over 2 km. per sec. greater.

The values of the rotational term and of the direction to the centre in the individual solutions necessarily have low weight on account of the large number of unknowns carried by relatively few stars and, for all the stars brighter than 5·5 magnitude, are practically indeterminate. The value of $\bar{r}A$ in the combined solution is obviously only a rounded average, although the direction to the centre seems reliable. The terms for the galactic rotation were included in this solution to allow for this known systematic effect and in order to obtain the best value of the solar motion for the whole group. It is proposed to solve again for the rotational term and direction to the centre with the values of the solar motion obtained from the combined solution. These determinations should give better values of these unknowns than the previous solutions, and it is unfortunate that they could not be completed in time for this lecture. As there have also been received recently, by the kindness of the director and Dr. Wilson of the Dudley Observatory, the proper motions of 300 additional B-type stars fainter than those in the P.G.C., it is obvious that we have still a considerable amount of very interesting work ahead of us in further discussing the motions of the B-type stars.

Interstellar Matter.—A discussion of the high-temperature stars would be incomplete without some reference to one of the most important results derived from their investigation, the presence and properties of diffuse matter in interstellar space. Although full details are given in the paper by Pearce and myself in *Monthly Notices*, **90,** 243, in which it is perhaps worth noting that less than a dozen of the 261 interstellar velocities used were obtained elsewhere than at Victoria, I hope I may be pardoned for giving a brief summary of the most important conclusions. It seems probable that the results obtained from the early investigation of the O-type stars, in which they were shown to be in rapid motion in all directions through widely distributed diffuse matter, which appeared to be nearly stationary with respect to the stellar system, formed the basis of Eddington's masterly theoretical discussion of the physical properties of this diffuse matter. If this is the case, it is a striking example of a deduction of

Distribution of O to B5 Stars.



Galactic Coordinates.

	Apex.
O to B2 brighter than 5·51 . . .	○
O to B2 fainter than 5·50 . . .	○
B3 to B5 brighter than 5·51 . . .	●
B3 to B5 fainter than 5·50 . . .	●

Apex (all stars O to B5) . . . +

Standard apex . . . . . ○

great cosmic importance following closely upon accurate observational data obtained for an entirely different purpose.

If this early investigation may be considered as forming the observational foundation on which Eddington built his theoretical superstructure, I think the later observational evidence of the uniform distribution of this diffuse matter and its participation in the orderly and majestic rotation of the galactic system may justly be considered as the capstone and finishing course to the whole structure. It seems to me it has added greatly to our conviction of the objective reality of the diffuse matter in interstellar space to have demonstrated its uniform distribution throughout the stellar system, and to have shown that it shares in the most beautifully exact way in the differential rotation of the galactic system.

Conclusions.—The investigation of the high-temperature stars at Victoria has led to important advances in several directions. It has provided more definite information about the masses and luminosities of the O- and B-type stars, the stars of highest temperature, greatest mass, and highest intrinsic luminosity in the sky. It has shown that the Wolf-Rayet stars, the most puzzling objects in the sky, are somewhat fainter intrinsically, about eight-tenths of a magnitude, than the absorbing O-type stars, and that their unique emission spectra are probably produced by continuous emission of luminous matter from the stars. It has furnished the most convincing evidence yet brought forward of the objective reality of the differential rotation of the galactic system, incidentally providing a new and powerful method of obtaining the average distances of homogeneous groups of these very distant high-temperature stars. The analysis of the motions of these stars has shown that the mysterious K term, the excess positive velocity, definitely present only in these high-temperature stars, this apparent expansion of the O- and B-type system, is partly due to systematic group motions of recession of the brighter B-type stars in the southern sky, and partly to the Einstein red displacement of the spectral lines of massive stars. The definitive analysis of the solar velocity with respect to the high-temperature stars makes it over 2 km. per sec. greater than normal, and the position of the solar apex over 15° northward and eastward of the position determined from the naked-eye stars.

Finally, the investigation of the high-temperature stars first definitely revealed the wide distribution of diffuse matter in the stellar system through which these stars were moving in all directions, and later showed that this matter was uniformly distributed and that it shared very definitely in the differential motions produced by a rotation of the galactic system.

I have great pleasure in testifying to the effective collaboration of my colleague Mr. J. A. Pearce in the observation, measurement, and discussion of the B-type stars. I am greatly indebted to my son, Professor H. H. Plaskett, who initiated the spectroscopic investigation of the O-type stars, for many helpful suggestions and criticisms, and to Dr. C. S. Beals for the use of part of his work on the Wolf-Rayet stars.

Effective Temperatures of Stars. Second Paper. By Professor R. A. Sampson, F.R.S.

The present paper may be regarded as a continuation of an earlier paper—"Effective Temperatures of Sixty-four Stars"*—which itself followed a preliminary survey †—"On the Estimation of the Continuous Spectrum of Stars"—superseding it in several respects. In the present communication it has not been found necessary to alter the treatment, except in minor regards, and these being allowed for, the whole of the observations, dating from 1923 January to 1929 May, are now incorporated in one series which figures in the General List at the end of the paper. There are in all 80 stars. Excluding *Polaris* which is taken on every plate, and α *UMa.*, β *UMa.* which were taken each 30 times at different zenith distances in order to investigate the effect of the earth's atmosphere, there are in all 730 observations or a mean of 9·5 per star. This number appears to be sufficient to guard against large anomalies. As before, the exposures and developments were made by Mr. Storey, the reductions of the spectra to photograms and the preparation of the interpretation curves for each batch of plates by Dr. E. A. Baker.

Reference may be made to the earlier papers for details of treatment, but it will be convenient to recite the points summarily here, as an opportunity for reviewing the matters in which it was necessary to exercise a certain amount of judgment.

The stars run down to about mag. 3 and are therefore, for the most part, giant stars of high luminosity. There are, however, a few exceptions. The spectra are taken with a Merz objective prism of 12° angle, in front of the object-glass of a 6-inch photovisual telescope of 9-feet focus. The dispersion from (B) to (K) is 22 mm. Sometimes as many as 10 spectra are taken consecutively on the same plate; in all cases a full set includes *Polaris* taken first and last. Panchromatic plates were used, of various makes, sometimes hypersensitised in the red by washing in running water and drying before exposure. Most of these makes were very satisfactory, and no difference in the results appeared, except in the few cases when there was insufficient intensity in the record for convenient measurement.

In developing the plates standard circumstances were observed, by developing for a fixed time, with a standard developer, at a fixed temperature. It was found that any one batch of plates was sensibly uniform. For the purpose of deriving relative intensities of light from measures of the photographic deposit, out of each batch of plates one was exposed to measured lights, approximately monochromatic, of different wave-lengths, from which "interpretation curves" were derived, valid for that batch. These curves are used, as explained in my earlier paper, so as to relate log (intensity) in the abscissa with the ordinate registered in the photogram produced by running the spectral record through the Koch photometer.

The photographic theory upon which the procedure depends

* *M.N.*, **85**, 212, 1925. † *M.N.*, **83**, 174, 1923.

postulates nothing except that for standard development the blackening for any batch of plates and for any wave-length is a regular function of the log (intensity of light) and exposure time ; from this there results an expansion of the log (intensity) in a double-power series, in which certain coefficients enter. If we neglect all except the lowest powers we have the well-known Schwarzschild law ; the values of these coefficients, both the lowest and the subsequent ones, were examined by laboratory experiment. From these experiments, coupled with this theory, it appears that

$$\log (J_\lambda/J'_\lambda) - \log (I_\lambda/I'_\lambda)$$

is independent of all exposure times, where J_λ, J'_λ are the intensities for the same wave-lengths of the radiations of the star and some standard, say *Polaris*, and I_λ the intensity from the interpretation-curve relative to I'_λ, an arbitrary standard or zero. Hence the gradients of these two expressions with respect to λ are equal, and adopting Planck's expression for the intensity of black-body radiation in terms of wave-length and temperature, we get an easy solution of this rather refractory equation, between c_2/T for the star and c_2/T' for *Polaris* in terms of the gradient as deduced from plotting the whole of the measures for all wave-lengths for the star and *Polaris*.

In this connection I would remark that these considerations cannot be much abbreviated without omitting something essential ; thus the version of the same theory which Messrs. Greaves and Davidson published—some months after the appearance of my second paper, to which they do not refer—and which they have cited on each subsequent occasion as the foundation of the Greenwich work, is illegitimate, since it leaves out of account the necessity of eliminating the exposure times.*

Before the equation emerges in a solvable form certain considerations require to be made. Instrumental absorption is eliminated completely by the method of treatment, but atmospheric absorption must be dealt with carefully. Before adoption the gradient as read from a plotted chart must be reduced to value at the zenith. The zenith distance correction was derived in the earlier paper in a rough-and-ready fashion, its great effect not having been foreseen. For the present paper the derivation was more careful and is given below. When found, it was applied to all the observations of the earlier series before they were incorporated with the present ones.

The treatment of atmospheric absorption resulting in a zenith distance correction, as given below, must be regarded still as preliminary. The effect is of at least as great importance in questions of intensity as atmospheric refraction in questions of position. It is, moreover, highly variable and its elements are insufficiently known.

* See *M.N.*, **86**, 33, 1925 ; **87**, 352, 1927 ; **90**, 104, 1929. The coefficients *a*, *b*, *c*, which they introduce in the expression $E = a + bD + cD^2$, are presumably functions of the exposure times, and are different from the pairs E_1, E_2 and E_3, E_4. Though no mention is made of their exposure times, I judge them to have been different, from their plate No. 6 (*M.N.*, **87**, 354) where δ *Per.* ($3^m.1$) and α *Cyg.* ($1^m.3$) are shown with spectra of nearly the same intensity.

Hitherto it has only been investigated carefully in connection with the solar constant, where the problem is special and somewhat different. If measures of intensity are ever to reach a standard of high precision, it will require the same thorough and mature treatment that has been given to atmospheric refraction.

When the radiation of each star is reduced to its zenith value it is assumed, tentatively for each wave-length, that on any one night the reduction of intensity is the same for all stars. This is incorrect. The transparency of the atmosphere for any wave-length may change rapidly and considerably before the exposures upon a plate are completed. It does so in an irregular manner. Moreover, there is evidence, rehearsed below, that systematically the second observation of *Polaris* on each plate shows more blue transparency than the earlier one. There seems no way at present of dealing with these features, except by comparing a star with the measures of *Polaris* interpolated for the moment of its observation, and by leaving any error that may be involved to be wiped out by accumulating sufficient observations of the star. This procedure has been followed. Another feature is certainly present, and if less important than atmospheric absorption, it is also less easy to determine and to eliminate, because its effect is systematic. In the previous paper I have called this the "pressure feature," because it is dependent upon the state of the star's atmosphere; I propose to change this name to the "disc effect." The disc effect for the case of the Sun is known and was calculated in the first paper from Abbot's data. Because light from the outer rings of the disc passes through a greater thickness of the Sun's atmosphere than that from the centre, it appears redder, and because the area of these rings is relatively large, the integrated effect is very considerable. We are wholly without observational material for any other star. It will, however, be conceded that for stars of earlier type the spectra are of a character that show a less dense absorbing atmosphere, so that the disc effect diminishes as the temperature rises. No other knowledge is at present available. Now for the Sun's whole disc the absorption increases as the wave-length diminishes. If, failing any other rule to carry us from the Sun to any other star, we adopt the simple one that the effect is the same for λT = const., *i.e.* that the disc effect, or pressure effect, may be represented by a factor $\psi(\lambda T)$, for any star, say the Sun, then we find that for the same wave-length the effect in a star of greater temperature is less, from which rule we get a figure for the disc effect for a star of any type which is at any rate of the right order and in the right direction. Though this procedure cannot claim very good foundations, I have thought it best to preserve it, as I did in the two earlier papers, as a token, representing by a not unreasonable amount an effect that is certainly there. The disc effect, if calculated by this rule, roughly comes out as an increase of from 200° to 300° to the temperature indicated by the uncorrected gradient, throughout—thus producing a change which is substantial in the lowest temperatures and negligible in the highest. Hence the temperatures recorded below are supposed to represent the centre of the star's disc and are about

200° above the values for the whole disc in K stars and 300° for A stars.

A further consideration of the disc effect raises the question whether it is dependent upon absolute magnitude. It is very improbable that a giant and a dwarf star, though showing the same spectral type, possess atmospheres that behave in the same way when the light is integrated over their discs. In this case the difference, if any, can be examined. In fact the colour index makes it certain that a dwarf star shows proportionately more blue light than a giant star of the same spectral class. This would reveal itself in a temperature investigation by the dwarf stars appearing on the whole systematically hotter than the giant stars of the same class. In my last paper I did not find such an effect. But there are no true dwarf stars in my list except *Procyon*. Later a collection of gradients, plotted against absolute magnitudes for each spectral class, will be found (p. 651). The indications are necessarily not very clear, but so far as they go they are consistent with a change of temperature following absolute magnitude, which is of the right order, and in the known direction, at any rate for those classes where the giant-dwarf fissure exists.

I now come to the consideration in detail of different points arising.

The Material.

Specimens of the photograms are given in the two previous papers. These may be taken as characteristic of the whole material. The summary outcome of each photogram, when measured and compared with measures of *Polaris*, is recorded as a gradient in the *General List*. Their degree of accordance may be judged by reading this list. Beyond, and before this, is the question how far the chart of the comparison, from which the gradient was derived, is consistent with a straight line, as it would be, nearly, if the observations were perfect and if the stars radiated as black bodies. Specimens of charts, distinguished as "normal" and "abnormal," are given in *M.N.*, **85**, 242, 243, 1925. These again may be taken as characteristic of limiting modes, the former lying very fairly on a straight line, and the latter offering problems of greater or less obscurity. In view of the large number of photograms I have measured and charts I have read, I would offer here a few remarks upon what they show.

The question of physical interest is, In what sense and how far do the stars radiate as black bodies? But on looking into this question more closely, it will be seen that it can only have a conventional answer.

Take the Sun, for instance. At the photosphere and at the centre of the disc, the quality of the radiation does not fit black-body emission, being consistently too great in the red and deficient in the violet. It is as nearly as possible certain that this failure is merely produced by the outer layers of the Sun's body, and that if we could penetrate a certain depth within, we should find true black-body radiation, at a higher temperature. In the same way the photograms of a star's radiation—allowing for the peculiarities of photographic rendering—can be con-

strued as true black-body radiation, but to do so presumably we must go outside the summits of the photogram, and further outside in the violet than in the red. Thus it becomes a conventional question.

Now it is fairly certain that there is a certain amount of general absorption in the star's surroundings, and more in the case of "later" type-stars than of "earlier." One cannot examine these photograms without feeling it. Regions appear to be suppressed from what we would judge to be a smooth development through neighbouring summits. But it is not safe to act upon this. Hence in measuring the photograms I have aimed at running a smooth curve through the summits, merely filling up the known or obvious absorptions.

We may conventionally define the curve so taken as representing the photosphere of the star, and the question the charts can answer is, Does this conventional photosphere radiate as a black body? The general answer is in the negative.

I have revised the charts after completing the discussion and reconsidered my impressions while measuring them. The quality of any one set may be judged quite fairly from the degree of accordance of the gradients recorded for each star in the *General List*. But systematically, stars "earlier" than *Polaris* show a stronger emission in the violet than belongs to black-body radiation, and stars "later" a weaker emission. This feature is so general that it may be taken as a rule, and the gradient has been determined while expecting it.

Of other systematic features I hesitate to affirm a physical reality. One is very frequent, namely an oscillation to and fro across the gradient line; for early stars the chart runs below the gradient line about λ 4700 and above about λ 5200, reversing this sequence in the later stars, though not in so marked a degree. But the feature is not constant, and appears to be chiefly due to treatment in reading, or defects in the material.

Of individual peculiarities in the stars, the information conveyed by the measures must not be pressed beyond the determination of gradients as given. This is all that was contemplated, and we approach too closely the range of accidental anomaly if we go further. I have examined the special cases selected for remark on p. 653 below, without noting anything except the brief remarks given there. The charts of these remarkable cases are, in all instances, ordinary to good.

Polaris.

A spectrum of *Polaris* is taken first and last on each complete plate. For purposes of control the second has been treated as if it were a different star, and the gradient derived by plotting the difference of log (intensity) as measured for different wave-lengths. The result in the sense "Last *minus* First" is shown in Table I.*

Thus, out of 77 observations, 13 are positive and 64 negative, the mean being $-\cdot084$. The earlier paper showed the same feature; out of 47, 10 were positive, 4 zero, and 33 negative, with a mean of $-\cdot077$. The conclusion apparently is that an ordinary night grows more transparent

* Negative results are marked by different type, and set to the right.

TABLE I.

Polaris. F8 : F9. Zenith Distance 34°.

Plate.	Observed Gradient "Last *minus* First."		Make and Batch.
25/2		·06	Imp. 4585.
/3		·10	,,
/6		·02	,,
/7		·14	,,
/8		·13	,,
/9		·25	,,
/10		·14	,,
/15		·15	Well. 9971 J.
/16	·02		,,
/17	·14		,,
/18	·08		,,
/21		·16	Imp. 4866.
/22	·28		,,
/23	·07		,,
/24	·04		,,
/25		·06	,,
/26		·28	,,
/28		·14	,,
/29		·42	,,
/30		·09	,,
/31		·31	,,
/32		·02	,,
/34		·15	,,
26/1		·01	,,
/1		·10	,,
/2		·04	,,
/4		·01	,,
/6		·10	Imp. 4881.
/7		·05	,,
/8		·02	,,
/9		·10	,,
/10		·08	,,
/11		·08	,,
27/3		·08	,,
/6	·04		,,
/7		·04	,,
/11		·10	,,
/13		·10	,,
/16		·15	,,
27/18		·17	Imp. 4881.
/19	·03		,,
/24		·19	,,
/26		·04	Imp. 5737.
/29		·07	,,
/34		·06	,,
/37	·02		,,
/39		·04	,,
28/3	·20		,,
/4	·04		,,
/6		·10	Imp. 5841.
/7		·06	,,
/8		·03	,,
/13		·21	,,
/14		·02	,,
/18		·10	Ilf. S.G. 6150A.
/21		·05	,,
/23		·07	,,
/24		·14	,,
/27		·05	Ilf. S.G. 6155A.
/28		·08	,,
/32		·10	,,
/34		·10	,,
/35		·11	,,
/36		·16	,,
/37		·22	,,
29/3		·12	,,
/4		·10	,,
/5		·28	,,
/6		·31	,,
/8		·10	,,
/9		·16	,,
/10		·14	,,
/11	·02		,,
/13	·18		,,
/14		·16	,,
/16		·27	,,
/18		·17	,,

to the blue in the course of the two hours or so required to take the plate. It is hard to see whether this should be ascribed to the lower strata or the upper strata of the Earth's atmosphere. The observations were taken at all hours of the evening, generally before midnight and not with any relation to sunset, so that the upper atmosphere would seem to be excluded. The effect may be due to a settling of dust out of the lower atmosphere, but for this it seems rather large. The feature has been dealt with in the same way as in the earlier paper, by comparing the measures for any star with the interpolated value at the time of its observation of the measures for *Polaris*. If we remove the mean value $-\cdot084$ from the observations the mean residue is $\pm\cdot076$, which may be taken as indicating the mean error of a gradient, derived from one observation.

Zenith Distance Correction.

Let the proportionate reduction of intensity in passing vertically through a layer of the atmosphere of thickness $d\zeta$, at height ζ above the surface of the Earth, be

$$1 - \sigma[\zeta]d\zeta/d,$$

where σ is a coefficient of absorption and d is a constant. Then if the ray arrives at the point of observation at an angle from the zenith z, we must replace this expression by

$$1 - \sigma[\{\overline{a + \zeta}^2 + \zeta^2 \tan^2 z\}^{\frac{1}{2}} - a] \sec z d\zeta/d,$$

where a is the radius of the Earth—the bending of the ray by refraction being ignored ; or say, by

$$1 - \sigma[\zeta + \tfrac{1}{2}\zeta^2 \tan^2 z/a] \sec z d\zeta/d.$$

Supposing σ has a constant value σ_0, up to a certain height h, and is thereafter zero, the whole reduction of intensity is

$$\begin{aligned} \Pi(1 - \sigma_0 \sec z d\zeta/d) &= \exp(-\sigma_0 \sec z h/d) \\ &\doteqdot 1 - \sigma_0 \sec z h/d. \end{aligned}$$

We may put $h/d = 1$ without loss of generality.

If σ is not a constant, it is evident that $\sigma_0 \sec z$ in this expression must be replaced by terms of the form

$$A \sec z + B \sec z \tan^2 z \ldots$$

where the constant B is of the order of the square of what may be called the " equivalent " height of the atmosphere, $\div a$. If there is evidence of a sensible value to the term B—such as appears below—it proves that sensible absorption takes place at a height which is some not inconsiderable fraction of the Earth's radius. It is, of course, only for large zenith distances that this will be noticeable.

Since the coefficient of absorption is zero in interstellar space, σ must at some height decrease, so that we may expect the sign of B to be opposite to that of A. To find the numerical values, the observations

were arranged so as to take α *UMa.* (Ko) and β *UMa.* (Ao) at a variety of zenith distances. In all, 30 observations of each were obtained, including those of the earlier series. It proved, however, that the

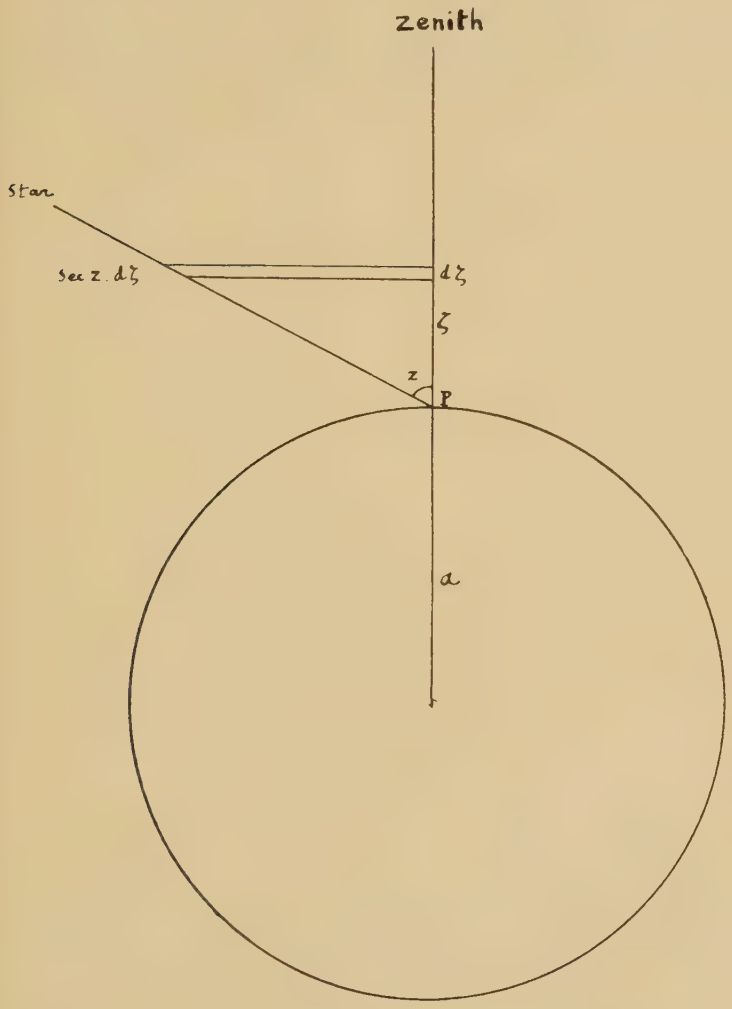


Fig. 1.

dispersion of the gradients found—presumably owing partly to variability in atmospheric absorption—was so considerable that the values indicated for the latter star were supplemented by plotting also all stars marked Ao in the Harvard scale. The mean measured gradient was plotted against the mean sec Z.D. As will be seen from inspection of

figure 2, the indication of the coefficient thus became quite clear, as well as consistent with that shown by the star β *UMa.* The Ao stars plotted along with β *UMa.* may not be all of the same class with it, but they seem to be sufficiently near to work jointly. As regards the Ko star, α *UMa.*, the classification is much less distinct. Those marked Ko in the Harvard scale are widely scattered in the Mount Wilson scale. A similar procedure was attempted for these stars, but it did not prove of value. In the end, the Ao stars alone were relied upon, and the

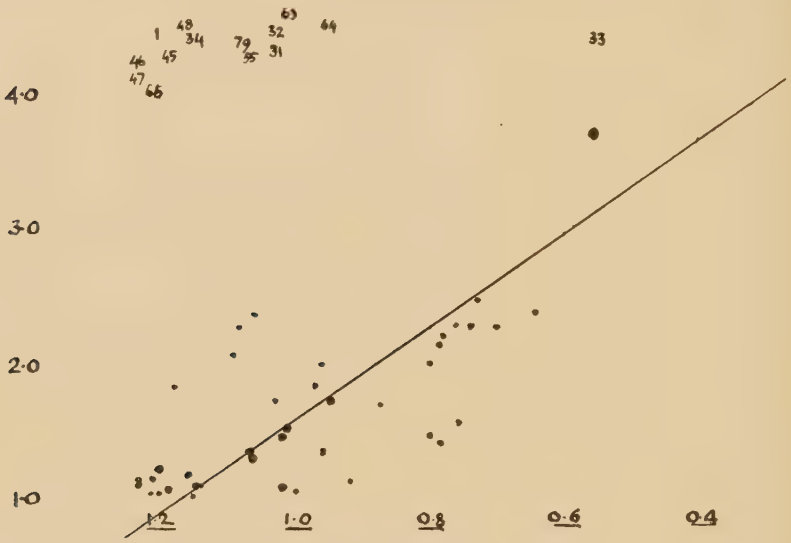


FIG. 2.

Ordinates : sec Z.D.
Abscissæ : gradients.
● Ao stars as numbered.
• β UMa.(Ao).

observations of the Ko star, α *UMa.*, are simply recorded as contributory evidence.

Recalling that the measured gradient is a gradient relative to *Polaris*, we read from the plotted observations of the Ao stars

$$\text{gradient} = -1{\cdot}15 + 0{\cdot}30(\sec z - \sec Z)$$

where Z is the zenith distance of *Polaris*. This is sufficient for all cases in this collection, with one exception. For *Sirius* the zenith distance is necessarily large. The mean sec Z.D. is 3·67 = sec 74° 12′, and the mean gradient out of 17 observations is − 0·59. Writing

$$-1{\cdot}15 + 0{\cdot}30(\sec z - \sec Z) + B(\sec^3 z - \sec^3 Z) = -0{\cdot}59$$

we find

$$B = -{\cdot}0046.$$

In a small number of cases outside those considered here, the second term requires to be taken into account. Thus I have applied in general the correction

$$-0{\cdot}30(\sec z - \sec Z) + {\cdot}0046(\sec^3 z - \sec^3 Z)$$

to the observed gradient, in order to reduce to the zenith value. But it must be noted that a good determination of the value of the coefficient

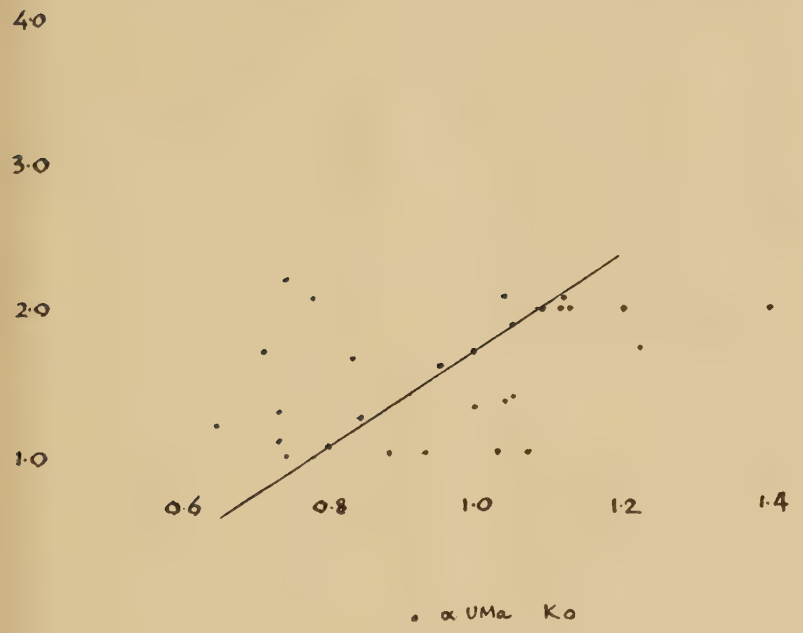


Fig. 3.

Ordinates: sec Z.D.
Abscissæ: gradients.

B requires more agreement on the spectral classification than exists at present.

In the *General List*, Table IX., the reduced mean given after the individual observations is derived from the mean of the observed gradients by applying this correction, *sec z* being taken as the mean of the secants of the individual zenith distances.

Determination of a Standard Temperature.

Logically, the method of obtaining the temperature by measuring the relative gradient for two stars is incomplete. It must be supplemented by fixing the standard temperature of one star of the series, after which the temperatures of the others follow. I proceed to review the possible steps.

Wilsing, and subsequently Greaves and Davidson, attempted to secure a terrestrial standard by comparing their spectra directly, or by help of some common link, with an arc at a temperature supposed known. This method may be considered as tried out and found unsuitable. It is probably answerable for both systematic and individual errors of scale in Wilsing's work, and the Greenwich observers have now given it up. Nevertheless it may not be superfluous to indicate the objections to which it is open. First, atmospheric absorption affects the star and the terrestrial source differently, and must be allowed for differentially. Then, any terrestrial temperature lies on the border of the lowest of the stellar temperatures, so that extrapolation is inevitable. Further, this extrapolation is unfavourable, because as will be seen by inspecting Table VIII. a small change in the lower temperatures produces a larger and progressive change in the rest of the series. Again, any direct comparison with a terrestrial standard is an attempt under unfavourable circumstances to make a direct determination of radiation constants, which even under the best circumstances of laboratory control are difficult to determine and somewhat discordant.

We may avoid these difficulties altogether if we can determine the temperature of one star of the series. The only star upon which direct and detailed observations are possible is the Sun. Even for the Sun the different methods give somewhat conflicting determinations, which do not closely follow the black-body law. Still they do not admit much departure from a temperature of 6000° A., or a little higher, for the disc as a whole. But the Sun does not figure in our present list, nor any star quite like it. The most convenient standard is *Capella*, which is nearly of the same spectral class. If *Capella* is accepted, we can pass from it to *Polaris*, and thence to the rest of the series. This was the method of my first paper. But because *Capella* is a giant and the Sun a dwarf, we should expect some reduction of the temperature of the latter before it was applicable to the former. Thus in my first paper, though I did not recognise the giant and dwarf distinction, I assigned to *Capella* a temperature of 5500° on the grounds that the photograms appeared remarkably suppressed in the blue, and also from the fact that a higher adoption produced what seemed excessive temperatures in the " earlier " stars. My second paper repeated this value, and it reappears in the present one.

Judging by the lines of the spectrum, the Sun is classed as G0. In the same way *Capella* is classed G0 by the Harvard scale and G1 by Mount Wilson. Referring to fig. 4, it will be seen that No. 23 *Capella* falls upon the standard curve more nearly at G3. This would agree with a difference of temperature of 500°.

Again, by reference to fig. 6, where gradient is plotted against absolute magnitude for the different spectral classes, it will be seen that in passing from the Sun ($4^{m}\cdot8$) to *Capella* ($0^{m}\cdot1$) we should expect an increase of gradient of ·28, or a fall of temperature of 600°. Thus we may consider the value 5500° for *Capella* as fairly certain. It is evident that we cannot ignore some such reasoning as the foregoing. All methods take account of the known temperature of the Sun in some fashion.

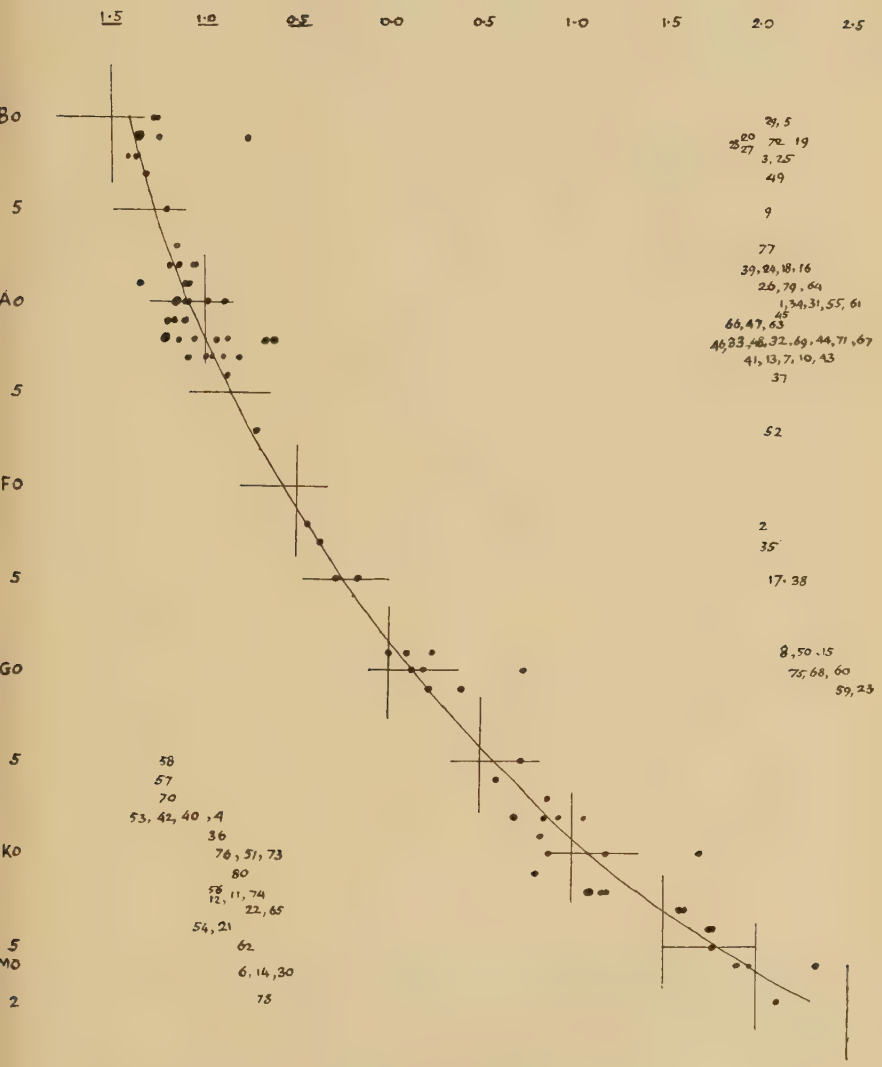


FIG. 4.—Observations and Standard Curve.

Ordinates: Mount Wilson spectra.
Abscissæ: observed gradients relative to Polaris.
Large dots indicate repetitions.

But the objection urged already to a direct comparison with the terrestrial arc, namely that at best it fixes a point upon the curve of temperatures which is unfavourable for determining the rest of the series, applies in minor degree to a determination by means of a star of class Go. There remains a good deal of admissible variety. Hence I proceed to review any other evidence for fixing a standard temperature, especially as it bears upon the temperatures of the earlier stars.

Russell, Dugan, and Stewart (*Astronomy*, II (1927), 753) collect a table of comparison of various methods of determination—colour index, heat index, energy curves, and diameters. The first gives the completest sequence, no other including the Bo stars. Their adoption virtually follows it. I have shown in fig. 5 a comparison between the results of this paper and their adoption. Their curve splits into two, giant and dwarf, from Go onwards. Since no true dwarfs occur in my list—unless *Procyon* is so called—we may leave the dwarf branch out of account in the comparison. The undetermined element in my work may be read as an arbitrary lateral shift of the two curves relative to one another. We note that it is possible to obtain a good agreement between the two curves from G5 to A5, but if this is done, my temperatures will fall for the higher temperatures below what they give. The two curves are systematically different in shape and cannot be made to agree by a choice of the adjustable parameter. The same is shown in the following table, where my curve has been shifted to agree with their temperatures at the lower end—the unit is 1000°.

Table II.

Comparison with Russell, Dugan, and Stewart.

	Temperature (Unit = 1000°).			Temperature (Unit = 1000°).	
	R., D., and S., p. 753.	Standard Curve.		R., D., and S., p. 753.	Standard Curve.
Bo	23·0	13·5	*g*Go	5·6	5·5
B5	15·0	11·9	*g*G5	4·7	4·8
Ao	11·0	9·4	*g*Ko	4·2	4·1
A5	8·6	8·6	*g*K5	3·4	3·4
Fo	7·4	7·5	*g*M2	3·1	3·1
F5	6·5	6·2			

We must choose between the two systems.

Contributory evidence is given by the comparison on p. 653 of certain stars used as standards by Greaves, Davidson, and Martin, which appear to confirm the continuation of my curve over the range A to B. Compare, for example, No. 3, γ *Peg.* B2, and No. 49, η *UMa.* B3.

In this position I submit that the authority of colour indices is hardly enough to enforce its acceptance. The values of temperature derived from it are calculated from a single formula, given by the authors (R., D., and S.) on p. 733, which is certainly a very good and convenient one, but depends for its numerical constants upon certain adoptions that were made without any idea of their use for determining temperature and can hardly claim to override a more direct procedure.

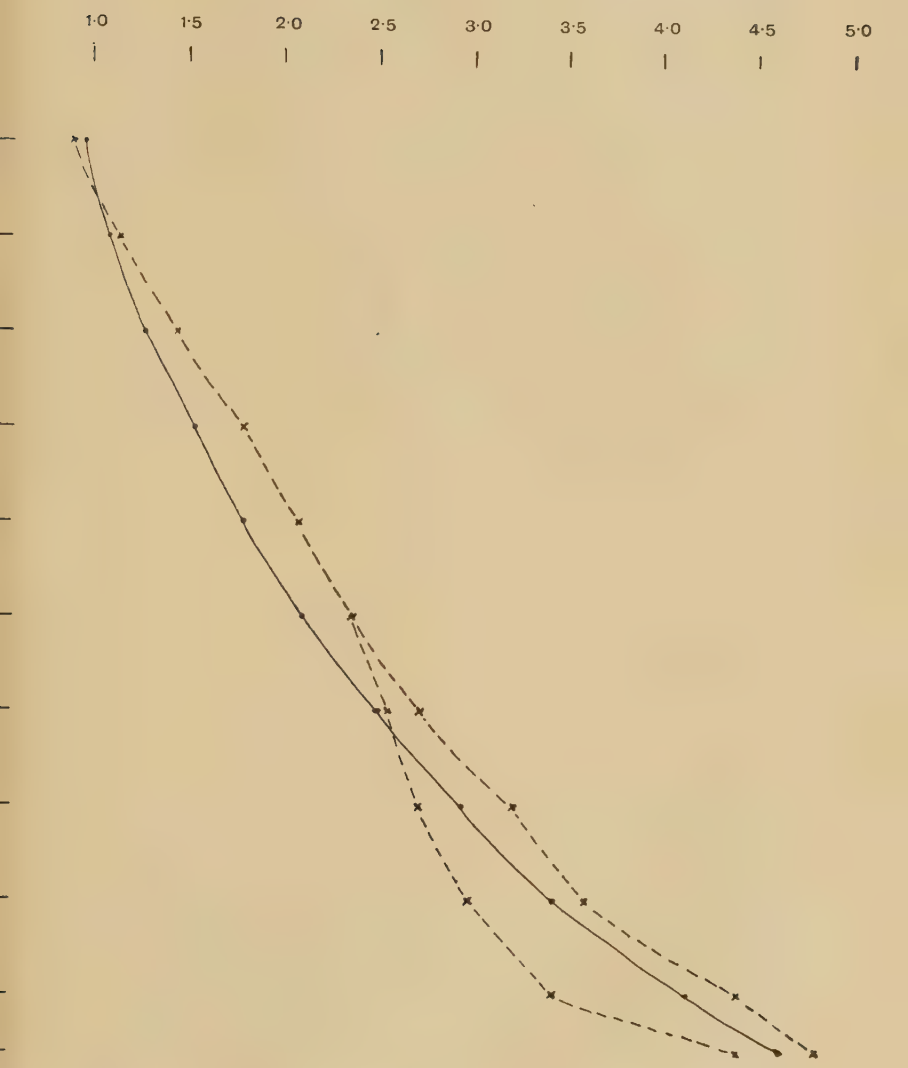


Fig. 5.—Comparison of Standard Curve with Russell, Dugan, and Stewart.

● Standard Curve.
× Russell, Dugan, and Stewart.
Co-ordinates as in Fig. 4.

Looking for further evidence for a standard temperature to adopt, we must add to Russell, Dugan, and Stewart's list the method originated by Saha and developed by R. H. Fowler and Milne,* of taking the marginal appearance, or the maximum strength, of certain spectrum lines. This method, again, cannot proceed without an adoption—in its case, the value of the "electron pressure." The actual adoption which Fowler and Milne make is intended to make their sequence run alongside other values of stellar temperature, especially in the middle and lower ranges. The point of special interest to the present discussion is that it indicates a high temperature for the early stars, viz. Bo – O9, 26,500°. Referring to the table above, we see that colour index gave 23,000°. On the whole I consider that other work makes it difficult to believe in a temperature for Bo stars which is not at least 20,000°, and this my curve would not allow unless Ao is about 13,000°.

Beyond this point I cannot find any grounds for a definite assertion. The actual adoption made in the following reductions is an absolute gradient for *Polaris* of 2·37, implying a temperature of 6500° A. The standard curve connecting gradient and spectral class, as indicated by these observations, is shown in fig. 4 and Table IV., p. 652, but it may be well to repeat it to a short extent here.

Table III.

	°		°
Bo	20,000	*g*G	6,200
Ao	12,600	Capella	5,500
Fo	8,600	*g*Ko	4,400
Polaris	6,500	*g*Mo	3,500

Gradient and Absolute Magnitude.

The way in which the measured gradients are distributed in relation to absolute magnitude is shown in fig. 6. There are not enough stars to make this figure demonstrative, for they must be grouped according to spectral class, and if we take together all stars from G6 to K5 and class them as K, they will cover a definite range of gradient, which indeed equals that shown in the figure. Hence one would hesitate to draw a definite conclusion from the figure, though I have shown, to the best of my judgment, what trend the actual observations express. It is perhaps not fallacious to remark that where the giant and dwarf fissure extends, namely over G, K, M, the slope shown agrees in sign and approximately in magnitude with the difference demonstrated by colour indices, being an increase of 0·08 gr./mag. from dwarf to giant.

The Standard Curve and Particular Cases.

The summary outcome of all the observations may be represented in fig. 4, where the mean observed gradient is plotted against the Mount

* Cf. *M.N.*, **84** (1924), 367, 499.

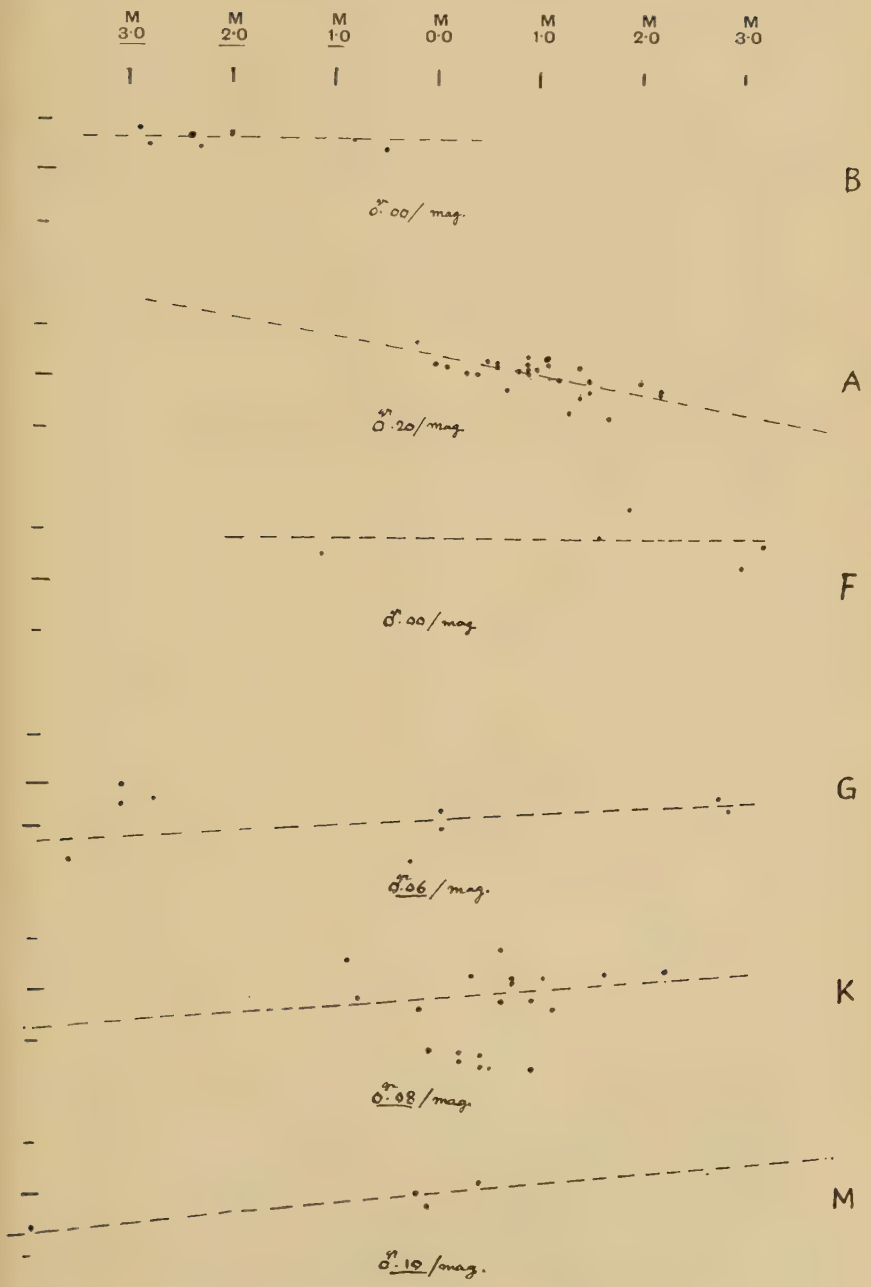


FIG. 6.—Gradient and Absolute Magnitude according to Spectral Class.
(Figures underlined are negative.)

Wilson spectrum for each star. The reference numbers of the stars are given alongside. In my former paper, I plotted the gradients against the Harvard classification also, but though I have done it here too for study, and I am not satisfied that the Mount Wilson system is in any sense final. I have not thought it useful to show it. I do not propose to discuss the spectral classification here. I made some remarks upon it in my earlier paper. I think it will be generally agreed that in experienced hands, such as those that have drawn up the existing systems, a consideration of the lines is a very safe method. Still we must recognise that the two schemes offered present wide divergences.

I would repeat the view expressed in my earlier paper, that classification by measured temperature is at least as reliable as that by estimation of the lines, and is preferable as a system, because it depends only upon a single variable.

The first step in discussing the massed observations is to ask what mean curve they indicate. I have shown this on the figure. Reduced to a table, and reckoning 2·37 as the absolute gradient of *Polaris*, as explained in a previous section (p. 650), we find the following values. Those in my earlier paper (*M.N.*, **85**, 237, 1925) are added for comparison.

Table IV.

		(*Cf.* 1925.)			(*Cf.* 1925.)
Bo	20,000°	(25·0)°	F6	7,100°	(7·2)°
2	18,300	(20·0)	8	6,700	(6·7)
4	16,700	(17·3)	Go	6,200	(6·2)
6	15,200	(15·6)	2	5,800	(5·8)
8	13,800	(14·3)	4	5,400	(5·3)
Ao	12,600	(13·1)	6	5,000	(4·9)
2	11,600	(12·1)	8	4,700	(4·5)
4	10,700	(11·2)	Ko	4,400	(4·2)
6	9,900	(10·3)	2	4,100	(3·9)
8	9,200	(9·5)	4	3,800	(3·6)
Fo	8,600	(8·9)	5	3,600	(3·5)
2	8,000	(8·3)	Mo	3,500	(3·4)
4	7,500	(7·7)	2	3,200	(3·2)

The earlier curve is a little steeper. A part of the difference is due to a reduction of c_2 from 14,600 to 14,300. It must be remembered that the new collection is of much greater weight.

Considering now the departure of individual dots from the standard curve, if we inspect the original measures of gradient shown for each star in Table IX., I think it will be generally agreed that the mean of ten observations of a star should seldom be 0·10 in error, whereas there are many cases in the figure that show a greater amount, and indeed more than twice the amount. Two possibilities arise. If we decide that

these departures are not merely accidental, must they be ascribed to the difficulty of producing a satisfactory spectral classification by lines which shall also agree with the classification by temperature, or are they due to individual peculiarities of the separate stars ?

I select the following cases for remark, being those in which the measured gradient for the star departs by $\pm$ ·25 or more from the point of the standard curve corresponding to its Mount Wilson spectrum.

Table V.

No.	Star.	Spectrum.	Abs. Mag.	Gradient.	*Cf.* Curve.
19	ζ Per.	B1 :	..	*·76*	*1·33*
30	α Ori.	Mo : Mo	*3·8*	2·33	1·96
60	β Dra.	Go : Go*p*	*3·5*	·73	·13
67	α Aql.	A5 : A2	1·7	*·62*	*1·00*
71	α Cep.	A5 : A2*n*	1·3	*·66*	*1·00*
73	ε Peg.	Ko : Ko	0·3	1·69	1·08
80	γ Cep.	Ko : K1	2·3	·80	1·21

In Nos. 30, 60, 80 the magnitude effect appears to be present ; if we allow for the high magnitudes of the first two and the low magnitude of the last, they would fall much nearer the curve. In one of these cases, No. 60, and also in No. 73 (*M.N.*, **85**, plate 9, p. 232), I showed photograms in a sequence of F, G, and K stars, illustrating their positions according to a temperature classification by the gradient, as being G5 and K4 respectively. If classified as above under Go and Ko the photograms appear to me to fall out of place.

In the case of Nos. 67 and 71 the Harvard classification would much diminish the departure. Finally No. 19 is already known as a star of low temperature for its class. (*Cf.* Greaves, Davidson, and Martin, *M.N.*, **90**, 111, 1929, where further reference will be found.)

Thus it seems certain that if a classification of spectra by lines is taken as the standard, no amount of re-observation of the individual stars will bring all stars of the same class to one point of any curve—there will remain individual differences, sometimes very large as in the cases of ζ *Per.*, β *Dra.*, and ϵ *Peg.*, but even as striking in the very numerous cases of minor but sensible departures, in excess of any error that can be admitted.

It will be seen that with very few exceptions the spectra of the B and A classes which are marked *s* lie to the left, *i.e.* are hotter than the standard, while those marked *n* are colder.

A certain number of stars in this list are included in a recent paper by Greaves, Davidson, and Martin, " Colour Temperatures of Twenty-four Stars, Suitable for Use as Spectro-photometric Standards " (*M.N.*, **90**, 104, 1929). A comparison cannot be made directly, because in order to assign definite values to their measures they adopt the figure A0, 11,000°, from Russell, Dugan, and Stewart. But if my numbers are reduced to the same datum we obtain the following parallel determinations :—

TABLE VI.

Comparison with Greaves, Davidson, and Martin.

(Unit, 1000°.)

No.			G., D., and M.′	*Cf.*
1	α And.	Ao : A1	11·6	11·9
3	γ Peg.	B2 :	15·2	15·2
7	δ Cas.	A5 : A3	8·8	9·9
10	β Ari.	A5 : A3	8·5	9·3
13	β Tri.	A5 : A3	8·8	10·1
18	η Tau.	B5 : B8	10·9	10·5
26	β Tau.	B8 : B9	12·3	13·8
31	β Aur.	Ao : Ao	10·1	10·0
34	α Gem.	Ao :	9·7	11·0
39	α Leo.	B8 : B8	11·5	11·7
41	β UMa.	Ao : A3	10·8	10·8
43	δ Leo.	A3 :	8·6	8·9
47	α CVn.	Ao : A1	11·1	11·6
49	η UMa.	B3 :	13·5	13·4
55	α CrB.	Ao : Ao	10·5	10·8
61	α Oph.	A5 : Ao	8·4	9·4
63	α Lyr.	Ao : A1	10·5	11·0
64	ζ Aql.	Ao : B9	10·4	10·8
67	α Aql.	A5 : A2	8·0	8·0
69	α Cyg.	A2 : A2	8·6	9·7
71	α Cep.	A5 : A2	7·7	8·2
79	α Peg.	Ao : B9	11·2	11·0

Besides the above standards there are the following stars given :—

TABLE VII.

No.			G., D., and M.	*Cf.*
5	γ Cas.	Bo :	10·0	12·6
16	β Per.	B8 :	11·1	11·3
19	ζ Per.	B1 :	8·3	8·7
20	ε Per.	B1 : B1	13·5	14·0
44	β Leo.	A2 : A2	9·4	9·3
45	γ UMa.	Ao : Ao	11·0	11·4
46	ε UMa.	Ao : A2	11·0	12·0
48	ζ^1 UMa.	Ao : A2	9·7	11·3
50	η Boo.	Go : F9	5·8	5·8
59	ζ Her.	Go : G1	5·4	5·5
66	δ Cyg.	Ao : A1	11·2	11·9

There are a certain number of differences, but considering the difference of method, the agreements appear to me to be more striking. It will be noticed that No. 3 and No. 49 or Nos. 5, 19, and 20 give little indication of reaching the temperature 23,000° assigned to Bo by the method of colour index.

General Table.

The General Table IX. which follows shows the separate stars in detail. Following the denomination of each star are given : Spectra on H.D. and M.W. scales ; visual magnitude and absolute magnitude, the latter taken as a rule from Adams's determinations ; then the actual observations taken, with reference number of the plate, the zenith distance, and the measured gradient relative to *Polaris* ; at the foot the reduced mean gradient after correcting for zenith distance ; and the temperature inferred. The last was taken from the same table that was employed in the previous paper (*M.N.*, **85**, 218), except that the value $c_2 = 14{,}300$ was adopted. It will be convenient to repeat the table here (Table VIII.). The argument Φ is the gradient as defined in that paper (p. 217), adopting the gradient for *Polaris* 2·37, and the deduced temperature T is therefore corrected for the " disc effect " and so is about 200° or 300° above a value appropriate to the integrated disc.

TABLE VIII.

Gradient and Temperature.
$c_2 = 14{,}300$.

T.	$c_2/T.$	Φ.	T.	$c_2/T.$	Φ.
3,000°	4·767	4·98	12,500°	1·144	1·30
3,500	4·086	4·29	13,000	1·100	1·26
4,000	3·575	3·78	13,500	1·059	1·22
4,500	3·178	3·37	14,000	1·021	1·19
5,000	2·860	3·05	14,500	·986	1·17
5,500	2·600	2·78	15,000	·953	1·14
6,000	2·384	2·57	15,500	·923	1·11
6,500	2·199	2·38	16,000	·894	1·09
7,000	2·043	2·21	16,500	·867	1·07
7,500	1·906	2·06	17,000	·841	1·05
8,000	1·788	1·93	17,500	·817	1·04
8,500	1·682	1·82	18,000	·795	1·02
9,000	1·589	1·72	18,500	·773	1·00
9,500	1·506	1·64	19,000	·753	·99
10,000	1·430	1·57	19,500	·733	·97
10,500	1·362	1·50	20,000	·715	·96
11,000	1·300	1·44	25,000	·572	·84
11,500	1·244	1·39	∞	0	·5
12,000	1·192	1·34			

TABLE IX.

General List.

No. 1. Boss 10.
α And.
Aop : Ain.

$2^{m}\cdot15$		$1^{M}\cdot1$
23/100	46°	*1·21*
24/144	33	*1·10*
/152	35	*·86*
25/28	29	*1·16*
/30	42	*1·16*
27/35$_1$	28	*1·46*
/35$_2$	29	*1·37*
28/3	35	*1·27*
/27	32	*1·24*
/28	28	*1·15*
		1·20
		T. 14,500°

No. 3. Boss 27.
γ Peg.
B2 : ...

$2^{m}\cdot87$		$2^{M}\cdot9$
23/100	45°	*1·56*
/113	43	*1·31*
24/158	41	*1·42*
25/22	44	*1·41*
/28	43	*1·35*
27/34	41	*1·36*
/36	44	*1·27*
28/24	41	*1·46*
/28	43	*1·40*
		1·42
		T. 20,000°

No. 5. Boss 199.
γ Cas.
Bop : ...

$2^{m}\cdot25$		...
24/100	32°	*1·98*
/113	13	*1·39*
/116	18	*2·12*
25/28	5	*·85*
/31	12	*·97*
26/11	5	*1·04*
27/31	30	*1·06*
/41	7	*1·30*
28/36	24	*·98*
		1·26
		(*a*) T. 15,500°

No. 2. Boss 12.
β Cas.
F5 : F2.

$2^{m}\cdot42$		$1^{M}\cdot6$
23/100	32°	*·56*
24/144	8	*·56*
/152	14	*·36*
25/30	25	*·36*
/31	8	*·64*
26/11	4	*·51*
27/2	45	*·35*
/31	25	*·41*
/37	16	*·46*
/41	7	*·73*
28/27	14	*·47*
/36	22	*·44*
		0·44
		T. 8000°

No. 4. Boss 135.
α Cas.
Ko : G8.

var.		$0^{M}\cdot7$
24/158	1°	1·32
25/31	14	·85
/33	3	1·03
27/2	23	·95
/37	8	1·06
/38	5	·94
/41	5	·59
28/36	20	1·38
		1·07
		T. 4400°

No. 6. Boss 259.
β And.
Mo : Mo.

$2^{m}\cdot37$		$0^{M}\cdot5$
23/101	39°	1·81
/110	26	2·21
/117	34	2·06
/122	33	2·33
24/158	20	2·15
25/31	23	2·08
26/6	37	2·00
27/2	36	1·62
/42	23	1·35
29/5	45	1·90
		1·89
		T. 3500°

Note (*a*).—No. 5. γ Cas. The first three observations are anomalous and probably excessive.

TABLE IX.—*continued.*

No. 7. Boss 314.
δ Cas.
A5 : A3*n*.
$2^{m}\cdot80$ $2^{M}\cdot0$

23/101	29°	·89
/124	7	1·06
25/28	9	·94
/31	15	1·08
26/11	6	·98
27/2	17	1·02
/37	9	·93
/41	5	1·20
28/36	22	1·00
		0·96
	T. 11,300°	

No. 8. Boss 325.
α UMi.
F8 : F9.
$2^{m}\cdot12$ $3^{M}\cdot0$
T. 6500°
See special list.

No. 9. Boss 419.
ε Cas.
B3 : B5*n*.
$3^{m}\cdot44$ $0^{M}\cdot5$

23/128	14°	1·54
24/131	16	1·27
26/3	7	1·03
/6	23	1·16
27/39	7	1·16
/42	15	1·43
28/35	8	1·27
/36	7	1·23
		1·21
	T. 14,700°	

No. 10. Boss 428.
β Ari.
A5 : A3.
$2^{m}\cdot72$ $2^{M}\cdot2$

23/101	55°	·74
/113	44	·73
24/158	37	·98
25/28	42	·79
/30	37	·95
27/6	44	·67
/7	43	·68
28/3	36	·96
/4	36	·93
/33	36	·98
/34	41	·90
		·89
	T. 10,700°	

No. 11. Boss 468.
γ^{1} And.
K0 : K2.
$2^{m}\cdot28$ $0^{M}\cdot1$

23/104	26°	1·28
/106	26	1·26
25/29	19	1·19
/31	22	1·04
27/2	25	·98
/25	35	1·02
/42	17	·90
28/3	16	1·36
29/2	25	1·28
/5	35	1·10
		1·18
	T. 4300°	

No. 12. Boss 477.
α Ari.
K2 : K2.
$2^{m}\cdot23$ $0^{M}\cdot7$

23/111	36°	1·22
/118	35	1·03
25/22	34	1·43
/29	35	·97
27/4	40	·94
27/6	40	·99
/7	37	1·26
28/3	33	1·02
/4	33	·99
/34	38	1·25
		1·10
	T. 4400°	

No. 13. Boss 482.
β Tri.
A5 : A3*s*.
$3^{m}\cdot08$ $1^{M}\cdot2$

23/126	23°	1·06
/128	23	·92
/131	25	·86
26/3	33	1·01
/6	39	1·04
27/39	31	·93
28/3	33	1·09
/37	32	1·10
		0·99
	T. 11,600°	

Table IX.—*continued.*

No. 14. Boss 691.
α Cet.
Mo : Mo.
$2^{m}\cdot02$ $0^{M}\cdot1$

	°	
23/111	57	2·54
/119	54	2·15
25/29	51	2·06
/32	52	2·28
27/7	54	1·65
/9	54	1·65
28/4	51	2·50
/34	51	2·39
/37	51	1·95
29/1	51	1·90
		1·97
		T. 3500°

No. 15. Boss 694.
γ Per.
F5*p* : F9.
$3^{m}\cdot08$ $0^{M}\cdot1$

	°	
23/129	11	·24
/131	8	·15
26/6	19	·13
/9	27	·10
28/11	29	·08
/35	13	·47
/37	12	·21
		0·24
		T. 5900°

No. 16. Boss 708.
β Per.
B8 : ...
var. $0^{M}\cdot4$.

	°	
23/110	28	·94
/111	18	*1·35*
25/4	34	*1·03*
/6	43	·97
26/1	5	·81
/3	5	*1·04*
27/2	13	·92
/4	15	·97
/7	17	·79
/42	13	*1·31*
28/3	13	*1·32*
/7	12	*1·20*
29/2	13	*1·05*
/5	23	*1·35*
		1·04
		T. 12,100°

No. 17. Boss 772.
α Per.
F5 : F5.
$1^{m}\cdot90$ $1^{M}\cdot1$

	°	
25/4	32	·22
/6	44	·06
26/1	6	·25
/3	6	·30
27·2	14	·30
/4	15	·25
/42	14	·54
28/3	12	·53
/7	12	·52
29/2	11	·17
		0·28
		T. 7400°

No. 18. Boss 869.
η Tau.
B5*p* : B8*n*.
$2^{m}\cdot96$ $0^{M}\cdot3$

	°	
23/129	32	·95
24/150	50	·73
/157	34	*1·09*
25/32	33	*1·00*
26/1	22	*1·00*
27/5	30	·85
/6	28	·85
28/4	22	*1·22*
/6	22	*1·12*
/7	23	*1·51*
29/5	30	*1·02*
/6	30	*1·10*
		1·05
		T. 12,200°

No. 19. Boss 894.
ζ Per.
B1 : ...
$2^{m}\cdot91$ ...

	°	
28/37	28	·78
29/1	30	·75
/2	24	·44
/4	26	·95
/5	25	·95
/6	27	·85
/7	25	·88
/8	32	·70
/9	35	·84
/10	38	·75
/12	39	·62
		0·76
		T. 9700°

No. 20. Boss 910.
ε Per.
B1 : B1*n*.
$2^{m}\cdot96$ $2^{M}\cdot0$

	°	
23/129	21	*1·23*
26/6	27	*1·28*
/9	31	*1·19*
/10	18	*1·55*
/11	24	*1·40*
/12	28	*1·22*
28/37	16	*1·45*
29/4	20	*1·41*

TABLE IX.—*continued.*

No. 20. Boss 910.—*contd.*

	°	
/5	18	*1·50*
/6	22	*1·41*
/7	18	*1·53*
/8	16	*1·47*
/9	31	*1·45*
/10	35	*1·52*
/12	35	*1·20*
		1·36

T. 18,300°

No. 21. Boss 1077.
α Tau.
K5 : K4.
1m·06 1M·0

	°	
23/119	45	1·95
24/157	43	1·91
25/30	42	1·90
/32	48	1·61
27/9	40	1·53
28/4	40	1·87
/7	44	2·06
29/5	44	2·06
/7	40	1·48
/9	47	1·69
		1·76

T. 3700°

No. 22. Boss 1167.
ι Aur.
K2 : K3.
2m·90 0M·0

	°	
25/6	45	1·60
/32	39	1·62
26/5	24	1·52
27/5	26	1·37
28/3	23	1·43
/6	24	1·52
/7	29	1·33
29/5	26	1·85
/6	31	1·99
/8	28	1·50
		1·59

T. 3800°

No. 23. Boss 1246.
α Aur.
G0 : G1.
0m·21 0M·1

	°	
23/942	47	·51
/943	43	·56
/103	44	·39
/115	43	·37
/118	43	·69
/123	43	·31
24/133	14	·21
/141	62	·73
/1484	60	·59
/1485	47	·46
25/1	24	·33
/2	12	·35
/12	43	·33
26/4	10	·34
/5	12	·33
27/11	20	·30
28/4	10	·51
29/3	11	·42
		0·40

T. 5500°

No. 24. Boss 1250.
β Ori.
B8*p* : . . .
0m·34 . . .

	°	
23/833	67	*1·02*
/834	67	*·89*
/835	67	*1·04*
/843	67	*·85*
/844	67	*·95*
/845	68	*1·11*
24/133	69	*·71*
25/2	65	*1·00*
/4	70	*·78*
/5	67	*·62*
/34	64	*·80*
26/4	64	*·78*
/5	65	*·81*
27/12	66	*·43*
/13	66	*·87*
/6	69	*·28*
28/8	66	*·72*
29/3	64	*·76*
/4	63	*·80*
		1·14

T. 13,400°

No. 25. Boss 1303.
γ Ori.
B2 : B2*s*.
1m·70 *2M·0*

	°	
23/88	57	*1·37*
25/5	54	*1·28*
/30	53	*1·13*
/34	50	*1·14*
28/6	50	*1·28*
/8	50	*1·25*
29/3	50	*1·20*
/4	50	*1·40*
		1·37

T. 18,500°

No. 26. Boss 1304.
β Tau.
B8 : B9*s*.
1m·78 *0M·2*

	°	
23/934	44	*1·59*
/935	44	*1·79*
25/30	33	*1·25*
/32	41	*1·30*

TABLE IX.—*continued.*

No. 26. Boss 1304.—*contd.*

	°	
27/5	29	*1·10*
28/4	27	*1·03*
/6	27	*1·33*
29/3	27	*1·36*
/7	27	*1·37*
/9	33	*1·56*
		1·35

T. 18,000°

No. 27. Boss 1339.
δ Ori.
Bo : B1*n*.
$2^{m}\cdot48$ $2^{M}\cdot4$

	°	
23/92	70	*1·42*
24/157	63	*1·14*
25/34	56	*1·12*
26/2	56	*1·20*
27/1	59	*·78*
28/6	56	*1·06*
/8	56	*1·20*
29/3	56	*1·01*
/4	56	*1·23*
		1·36

T. 18,300°

No. 28. Boss 1370.
ε Ori.
Bo : B1*n*.
$1^{m}\cdot75$ $2^{M}\cdot4$

	°	
$23/93_2$	72	*1·51*
$/93_3$	72	*1·27*
24/175	62	*·89*
25/34	57	*1·00*
26/2	57	*1·17*
27/1	61	*·82*
28/6	57	*·87*
/8	57	*1·20*
29/3	57	*1·05*
/4	57	*1·13*
		1·36

T. 18,300°

No. 29. Boss 1398.
ζ^1 Ori.
Bo : Bo*n*.
$2^{m}\cdot05$ $2^{M}\cdot8$

	°	
23/90	72	*1·08*
25/34	58	*1·02*
26/2	58	*1·08*
27/6	62	*·87*
/9	58	*·90*
28/6	58	*1·15*
/8	59	*·85*
29/3	59	*1·17*
/4	59	*1·12*
		1·27

T. 15,800°

No. 30. Boss 1468.
α Ori.
Mo : Mo.
var. $3^{M}\cdot8$

	°	
$23/88_4$	61	2·13
$/88_5$	61	2·13
25/12	68	2·95
/34	49	2·67
26/2	49	2·50
28/6	49	2·42
29/6	50	2·47
/9	51	2·67
		2·33

T. 3200°.

No. 31. Boss 1478.
β Aur.
Ao*p* : Ao*np*.
$2^{m}\cdot07$ $1^{M}\cdot5$

	°	
23/126	13	*1·02*
26/2	12	*1·10*
/6	13	*1·13*
27/5	13	*·92*
/20	36	*1·04*
28/9	20	*·96*
/10	15	*1·12*
/15	36	*·78*
29/6	16	*1·09*
		0·98

T. 12,100°

No. 32. Boss 1690.
γ Gem.
Ao : A2*s*.
$1^{m}\cdot93$ $0^{M}\cdot9$

	°	
25/3	40	*1·03*
/12	53	*1·01*
26/2	40	*1·04*
/8	40	*1·04*
27/5	40	*·93*
/12	46	*·91*
28/10	42	*1·37*
/14	48	*1·02*
29/6	40	*·84*
/7	41	*·87*
/11	40	*1·12*
		1·05

T. 12,300°

TABLE IX.—*continued.*

No. 33. Boss 172.
α CMa.
Ao : A2*s*.
$1^{m}\cdot58$ $0^{M}\cdot9$

	°	
23/85$_{1a}$	78	·60
/85$_{1b}$	78	·94
/87$_{3}$	75	·87
/87$_{4}$	74	·61
/87$_{5}$	74	·71
25/24	74	·65
/25	73	·50
/5	73	·55
26/6	73	·71
/8	73	·59
27/6$_{5}$	73	·19
/6$_{6}$	73	·30
28/8	73	·55
29/4	73	·44
		[1·17]
(*b*)		T. [13,800°]

No. 34. Boss 1978.
α^{2} Gem.
Ao : ...
$1^{m}\cdot90$ $1^{M}\cdot4$

	°	
25/3	15	1·07
/8	18	1·26
26/5	30	1·32
/8	17	1·16
27/13	15	1·15
/15	20	1·05
28/10	14	1·23
/11	15	1·30
/14	22	1·10
29/11	15	1·08
/10	37	·93
		1·11
		T. 13,000°

No. 35. Boss 2008.
α CMi.
F5 : F3.
$0^{m}\cdot48$ $3^{M}\cdot2$

	°	
23/85$_{3}$	52	·32
/85$_{4}$	52	·46
25/3	41	·20
/8	43	·29
26/8	43	·25
27/13	41	·68
/15	44	·09
28/10	41	·31
/14	46	·29
29/11	41	·20
		0·37
		T. 7700°

No. 36. Boss 2031.
β Gem.
Ko : G9.
$1^{m}\cdot21$ $1^{M}\cdot7$

	°	
23/81$_{4}$	35	1·06
/81$_{5}$	35	1·12
/82$_{2}$	33	1·15
/82$_{3}$	32	1·12
/82$_{4}$	31	1·27
25/3	28	·78
/8	29	·80
26/5	40	·65
/8	30	·62
27/13	28	·66
/15	32	·54
28/10	28	·68
/11	28	·56
/14	34	·86
29/16	48	·55
		0·83
(*c*)		T. 4800°

No. 37. Boss 2404.
ι UMa.
A5 : A4*n*.
$3^{m}\cdot12$ $2^{M}\cdot2$

	°	
24/134	8	·95
/136	16	·91
27/18	10	·96
/20	11	·83
29/7	29	·84
/8	20	·78
/9	17	1·20
/10	14	1·00
/11	17	1·00
/13	9	·71
		0·87
		T. 10,500°

No. 38. Boss 2552.
θ UMa.
F8*p* : F5.
$3^{m}\cdot26$ $3^{M}\cdot0$

	°	
24/135	9	·04
/136	20	·36
27/18	6	·37
/20	7	·27
29/7	26	·28
/8	19	·10
/9	13	·16
/10	11	·20
/11	16	·20
/13	7	·20
		0·17
		T. 7000°

Note (*b*).—No. 33. *Sirius* was used to determine atmospheric absorption.
Note (*c*).—No. 36. The early measures seem systematically greater; both plates 23/81 and 23/82 were taken on the same day.

TABLE IX.—*continued.*

No. 39. Boss 2698. α Leo. B8 : B9*n*.		
$1^{m}\cdot34$		$0^{M}\cdot5$
25/7	35°	*1·65*
/8	37	*1·35*
26/8	49	*·93*
/9	47	*·97*
27/3	61	*·77*
/10	36	*·89*
/13	39	*1·15*
/15	35	*1·21*
28/11	43	*1·56*
/13	39	*1·06*
/14	34	*1·09*
/15	34	*··92*
		1·18
	T. 14,000°	

No. 40. Boss 2742. γ^{1} Leo. Ko : G8.		
$2^{m}\cdot61$		$0^{M}\cdot8$
25/7	36°	1·08
26/9	50	·87
/10	36	·90
27/3	63	·92
/13	39	·97
/16	37	1·11
28/11	44	·95
/13	42	1·00
/14	36	·85
29/14	36	1·03
/15	36	1·00
		0·92
	T. 4700°	

No. 41. Boss 2930 β UMa. Ao : A3*s*.		
$2^{m}\cdot44$		$1^{M}\cdot0$
23/96	30°	*1·49*
/97	5	*1·41*
25/1	61	*1·09*
/9	15	*1·20*
/16	64	*1·08*
/25	64	*·70*
/29	57	*1·18*
/32	63	*·78*
/33_2	66	*·73*
/33_3	64	*·76*
26/1	54	*·87*
/3	62	*·79*
/4	45	*·79*
/7	50	*·76*
/11	57	*·97*
27/3	60	*·96*
/4	55	*1·03*
/7	42	*·96*
/8	40	*·96*
/11	27	*·92*
/16	14	*1·21*
/22	65	*1·06*
/32	64	*·74*
/34	65	*·64*
28/5	47	*·80*
/13	14	*1·00*
/14	7	*1·13*
/35	60	*·80*
29/8	25	*1·14*
		1·09
	T. 12,700°	

No. 42. Boss 2933. α UMa. Ko : G8.		
$1^{m}\cdot95$		$0^{M}\cdot4$
23/89	17°	1·03
/96	6	·58
24/148	61	1·07
25/1	54	1·00
/9	14	1·07
/16	60	1·40
/25	61	1·04
/29	54	·71
/32	60	1·13
/33_3	61	1·12
/33_5	60	1·09
26/1	52	·95
/3	58	1·05
/4	44	1·04
/7	45	1·05
/11	53	·83
27/3	63	·74
/4	35	·65
/7	40	·73
/8	38	·84
/11	27	·73
/16	15	·88
/22	61	·78
/24	63	1·12
/32	60	1·20
28/5	42	1·00
/13	16	·93
/14	6	·74
/35	55	1·22
29/8	23	·80
		0·85
	T. 4700°	

TABLE IX.—*continued.*

No. 43. Boss 2972.
δ Leo.
A3 : . . .
$2^{m}\cdot58$ $1^{M}\cdot4$

25/7	27°	·88
/11	34	·92
26/9	43	·76
/10	28	·63
27/3	64	·65
/13	32	·77
28/13	35	·97
/15	28	·74
29/17	25	·73
		0·81
		T. 10,000°

No. 44. Boss 3101.
β Leo.
A2 : A2*n*.
$2^{m}\cdot23$ $1^{M}\cdot5$

25/7	35°	·95
/11	40	1·02
26/10	35	·76
27/13	39	·85
/16	38	·89
28/13	37	·85
/15[a]	33	·84
29/17	30	·73
		0·87
		T. 10,500°

No. 45. Boss 3117.
γ UMa.
Ao : Ao*n*.
$2^{m}\cdot54$ $0^{M}\cdot9$

23/95	5°	1·21
/97	5	1·50
25/9	18	1·14
/11	18	1·28
27/11	30°	1·02
/14	14	1·07
/16	11	1·09
29/13	19	1·13
/14	30	1·30
		1·15
		T. 13,500°

No. 46. Boss 3363.
ε UMa.
Aop : A2*s*.
$1^{m}\cdot68$ $0^{M}\cdot9$

23/89_2	34°	1·23
/89_3	35	1·29
/95	7	1·44
25/9	22	1·28
27/11	35	1·02
/16	18	1·20
29/13	24	1·25
/14	35	1·12
		1·21
		T. 14,700°

No. 47. Boss 3370.
α CVn.
Aop : A1*s*.
$2^{m}\cdot90$ $0^{M}\cdot6$

23/98	20°	1·36
27/11	34	1·17
/16	25	1·24
/18	21	1·24
29/13	27	1·14
/14	36	1·25
		1·17
		T. 13,800°

No. 48. Boss 3474.
ζ^1 UMa.
Aop : A2*s*.
$2^{m}\cdot40$ $1^{M}\cdot1$

23/95	14°	1·34
/97	12	1·37
27/11	38	·99
/21	15	1·04
29/13	24	1·17
/14	39	1·05
		1·14
		T. 13,400°

No. 49. Boss 3566.
η UMa.
B3 : . . .
$1^{m}\cdot91$ $0^{M}\cdot8$

23/95	22°	1·55
/97	12	1·70
25/6	41	1·25
27/11	42	1·16
/21	19	1·00
29/13	29	1·36
/14	35	1·39
		1·32
		T. 17,000°

No. 50. Boss 3596.
η Boo.
Go : F9.
$2^{m}\cdot80$ $2^{M}\cdot8$

23/98	40°	·19
26/10	45	·18
27/18	33	·01
/19	37	·11
29/16	36	:07
/18	35	·14
		0·10
		T. 6300°

TABLE IX.—*continued.*

No. 51. Boss 3662.
α Boo.
Ko : Ko.
$0^{m}.24$ $1^{M}.2$

$23/92_3$	51°	1·22
$/92_4$	50	1·21
$/92_5$	48	1·57
$/94_4$	55	1·20
$/94_5$	54	1·51
25/10	42	1·28
26/10	43	·97
27/18	35	1·13
/19	39	1·25
29/16	37	1·23
/18	35	1·09
		1·18
		T. 4300°

No. 52. Boss 3722.
γ Boo.
Fo : A3*n*.
$3^{m}.00$ $1^{M}.9$

27/18	37°	*·92*
/19	40	*·74*
29/16	31	*·56*
/18	29	*·64*
		0·72
		T. 9400°

No. 53. Boss 3771.
ε Boo.
Kop : G8.
$2^{m}.70$ $0^{M}.8$

25/10	36°	·89
27/18	36	·48
/19	35	·69
29/16	31	·75
/18	30	·63
		0·69
		T. 5000°

No. 54. Boss 3809.
β UMi.
K5 : K4.
$2^{m}.24$ $0^{M}.5$

23/98	29°	1·86
/116	19	1·97
/122	20	1·82
27/21	22	1·80
29/18	20	1·42
/19	22	1·38
		1·75
		T. 3700°

No. 55. Boss 3961.
α CrB.
Ao : A*on*.
$2^{m}.31$ $0^{M}.9$

23/98	56°	*1·14*
25/10	46	*1·16*
/13	43	*1·14*
27/18	28	*1·26*
/19	30	*1·02*
29/16	28	*·78*
/18	27	*·98*
		1·09
		T. 12,800°

No. 56. Boss 4001.
α Ser.
Ko : K2.
$2^{m}.75$ $1^{M}.0$

25/13	62°	1·31
27/21	49	1·35
29/18	52	1·12
/19	52	1·20
		1·09
		T. 4400°

No. 57. Boss 4192.
η Dra.
G5 : G6.
$2^{m}.89$ $0^{M}.7$

23/107	32°	·54
/108	42	·72
27/19	29	·53
/21	29	·60
/27	39	·64
29/18	26	·43
/19	32	·59
		0·58
		T. 5200°

No. 58. Boss 4204.
β Her.
Ko : G5.
$2^{m}.81$ $0^{M}.2$

25/15	40°	·96
27/23	45	·60
/26	49	·78
/29	46	·66
28/22	50	·92
		0·72
		T. 4900°

No. 59. Boss 4246.
ζ Her.
Go : G1.
$3^{m}.00$ $2^{M}.9$

25/13	48°	·19
/15	44	·58
27/23	46	·20
/26	51	·23
/29	49	·16
28/22	51	·47
		0·22
		T. 6000°

TABLE IX.—*continued.*

No. 60. Boss 4443.
β Dra.
Go : Go*p*.
$2^{m}\cdot99$ $3^{M}\cdot5$

	°	
23/105	39	·49
/143	38	·84
/143	45	·70
27/24	35	·75
/25	48	·66
/28	34	·69
28/22	34	·93
/23	28	·83
/25	32	·83
		0·73
	T.	4900°

No. 61. Boss 4459.
α Oph.
A5 : A0*n*.
$2^{m}\cdot14$ $0^{M}\cdot7$

	°	
23/105	67	·72
24/145	67	·59
25/15	49	*1·09*
/16	44	*1·23*
27/23	51	·43
/24	46	·54
/26	50	·72
28/23	47	·45
		0·89
	T.	10,700°

No. 62. Boss 4541.
γ Dra.
K5 : K5.
$2^{m}\cdot42$ $0^{M}\cdot6$

	°	
23/99	20	2·19
/112	32	1·76
25/16	22	1·55
/21	36	1·68
27/24	29	1·62
/26	38	1·72
/28	26	1·59
28/22	25	1·73
/23	28	1·78
		1·76
	T.	3700°

No. 63. Boss 4722.
α Lyr.
A0 : A1*s*.
$0^{m}\cdot14$ $0^{M}\cdot6$

	°	
23/118	58	*·89*
/123	45	*1·05*
24/139$_2$	35	*·83*
/139$_3$	38	*1·00*
/139$_4$	42	*·94*
/139$_5$	45	*·91*
25/10	70	*·80*
/27	53	*1·09*
27/22	37	*1·33*
/27	43	*1·20*
28/18	28	*1·08*
/22	38	*·98*
		1·11
	T.	13,000°

No. 64. Boss 4858.
ζ Aql.
A0 : B9*n*.
$3^{m}\cdot02$ $0^{M}\cdot8$

	°	
23/103	57	*1·15*
/115	64	*1·30*
25/21	50	*·87*
/26	60	*·62*
27/23	50	*·78*
/26	52	*1·10*
/28	44	*1·02*
28/23	50	*·72*
		1·09
	T.	12,800°

No. 65. Boss 5047.
γ Aql.
K2 : K3.
$2^{m}\cdot80$ $0^{M}\cdot3$

	°	
23/105	52	1·73
/112	52	1·58
25/17	58	1·82
/21	52	1·45
27/27	48	1·58
/28	48	1·60
/30	49	1·44
28/18	48	1·62
/20	46	2·40
/21	49	1·84
		1·61
	T.	3800°

No. 66. Boss 5048.
δ Cyg.
A0 : A1*n*.
$2^{m}\cdot98$ $1^{M}\cdot1$.

	°	
23/103	32	*1·34*
/112	26	*1·35*
25/19	34	*1·33*
/23	39	*1·10*
27/27	30	*1·07*
/29	20	*1·21*
/34	21	*1·16*
28/23	24	*1·15*
		1·20
	T.	14,500°

No. 67. Boss 5062.
α Aql.
A5 : A2*n*.
$0^{m}\cdot89$ $1^{M}\cdot7$

	°	
25/17	61	*·42*
/20	58	*·28*
27/22	54	*·53*
/24$_2$	47	*·53*
/24$_3$	48	*·42*

TABLE IX.—*continued.*

No. 67. Boss 5062.—*contd.*

	°	
/30	53	·56
28/18	50	·55
/20	48	·58
/21	49	·58
		0·62
		T. 8800°

No. 68. Boss 5229.
γ Cyg.
F8*p* : G*op.*
$2^m.32$ $3^M.0$

	°	
23/99	17	·00
/117	23	*·05*
24/153	23	·12
/156	35	·11
25/17	33	·65
/19	35	·21
/20	30	·32
27/27	33	·02
/29	24	·03
/33	21	·04
28/26	24	·25
/27	21	·25
/29	27	·19
		·18
		T. 6100°

No. 69. Boss 5320.
α Cyg.
A2*p* : A2*s.*
$1^m.33$ ...

	°	
23/99	12	*1·25*
/117	22	*1·59*
/123	27	*1·28*
24/153	24	*·50*
25/17	30	*·92*
/23	35	*·70*
27/27	29	*·88*
/33	17	*·78*
/35	17	*·82*
28/26	19	*·83*
		0·93
		T. 11,000°

No. 70. Boss 5336.
ε Cyg.
K0 : G7.
$2^m.64$ $0^M.8$

	°	
24/150	27	1·00
/151	29	·68
/153	30	·79
25/18	25	·94
/19	36	·98
/20	39	·84
27/33	23	·72
/34	27	·63
/35	14	·79
28/26	27	·90
/29	29	1·03
		0·87
		T. 4700°

No. 71. Boss 5480.
α Cep.
A5 : A2*n.*
$2^m.60$ $1^M.3$

	°	
24/150	9	*·68*
/151	13	*·78*
25/19	21	*·67*
/23	23	*·83*
27/33	9	*·70*
/35	9	*·70*
/37	9	*·60*
28/25	6	*·71*
/30	9	*·72*
		0·66
		T. 9100°

No. 72. Boss 5532.
β Cep.
B1 : B1*s.*
$3^m.33$ $2^M.3$

	°	
23/121	25	*1·37*
/124	26	*1·25*
25/25	23	*1·05*
/26	18	*1·14*
27/35	16	*1·32*
/37	16	*1·34*
28/33	23	*1·43*
/34	32	*1·42*
		1·25
		T. 15,300°

No. 73. Boss 5584.
ε Peg.
K0 : K0.
$2^m.54$ $0^M.3$

	°	
24/153	48	1·68
/155	49	1·96
25/21	47	1·78
/24	47	1·70
27/28	46	1·58
/29	46	1·82
/30	46	1·64
28/28	46	1·89
/30	47	1·82
		1·69
		T. 3700°

No. 74. Boss 5714.
ζ Cep.
K0 : K2.
$3^m.62$ $0^M.5$

	°	
23/124	24	1·73
/125	22	1·69
25/26	16	1·36
/27	7	1·48
27/38	17	1·61
/39	11	1·68
28/31	20	1·55
/32	11	1·59
		1·63
		T. 3800°

TABLE IX.—*continued.*

No. 75. Boss 5807.
δ Cep.
Go : Go.
var. $2^{M}.7$

23/108	1°	·00
/109	8	·02
/123	8	·28
/125	10	·24
/127	8	·00
/128	8	·06
/130	13	·25
/131	27	·34
/132	10	·05
24/150	5	*·01*
/151	1	·18
/154	12	*·39*
/155	18	*·14*
		·13
(*d*)		T. 6200°

No. 76. Boss 5891.
ι Cep.
Ko : Ko.
$3^{m}.68$ $1^{M}.1$

23/132	19°	·93
24/143	15	·85
25/26	16	·62
/28	10	·90
27/35	10	·65
/37	10	·72
28/31	21	·82
/32	19	1·08
		0·87
		T. 4700°

No. 77. Boss 5933.
o And.
B3 : B7*n*.
$3^{m}.5$ $0^{M}.0$

23/132	27°	*1·05*
/143	18	*1·25*
25/26	16	*·95*
/27	16	*1·31*
27/39	18	*1·12*
28/32	20	*1·29*
/33	15	*1·37*
		1·15
		T. 13,500°

No. 78. Boss 5940.
β Peg.
Mo : M2.
$2^{m}.61$ $0^{M}.0$

23/122	50°	2·30
24/144	31	2·24
/151	30	1·79
/155	33	2·42
25/22	35	1·94
/24	28	1·96
27/29	28	2·17
/30	28	2·33
/34	28	1·80
28/24	29	2·07
/28	28	2·15
		2·11
		T. 3400°

No. 79. Boss 5944.
α Peg.
Ao : B9*s*.
$2^{m}.57$ $0^{M}.1$

24/154	43°	*1·13*
/155	43	*1·10*
25/22	44	*1·12*
/24	40	*·96*
27/30	40	*1·12*
/34	40	*1·14*
28/24	40	*·86*
/28	40	*1·16*
		1·11
		T. 13,000°

No. 80. Boss 6078.
γ Cep.
Ko : K1.
$3^{m}.42$ $2^{M}.3$

23/115	21°	·99
/121	21	·86
25/25	23	·76
/26	23	·64
27/39	23	·57
/41	21	·54
28/33	22	·77
/34	26	·94
		0·80
		T. 4800°

Note (*d*).—No. 75. δ Cep. See *M.N.*, **85**, 239, 1925.

On the Instability of Einstein's Spherical World.

By A. S. Eddington, F.R.S.

1. Working in conjunction with Mr. G. C. McVittie, I began some months ago to examine whether Einstein's spherical universe is stable. Before our investigation was complete we learnt of a paper by Abbé G. Lemaître * which gives a remarkably complete solution of the various questions connected with the Einstein and de Sitter cosmogonies. Although not expressly stated, it is at once apparent from his formulæ that the Einstein world is unstable—an important fact which, I think, has not hitherto been appreciated in cosmogonical discussions. Astronomers are deeply interested in these recondite problems owing to their connection with the behaviour of spiral nebulæ; and I desire to review the situation from an astronomical standpoint, although my original hope of contributing some definitely new result has been forestalled by Lemaître's brilliant solution.

Finitude of space depends on a "cosmical constant" λ, which occurs in Einstein's gravitational equations $G_{\mu\nu} = \lambda g_{\mu\nu}$ for empty space. On general philosophical grounds † there can be little doubt that this form of the equations is correct rather than his earlier form $G_{\mu\nu} = 0$; but λ is so small as to be negligible in all but very large scale applications. Except in so far as a value may be suggested by astronomical survey of the extragalactic universe, λ is unknown; or it would be better to say that we do not know the lengths of the objects and standards of our ordinary scale of experience in terms of the natural cosmical unit of length $1/\sqrt{\lambda}$. Besides involving λ, the shape and size of space depend on the amount of matter contained in the universe and the way it is distributed. Naturally space will only be of a perfectly spherical form if the matter (which by Einstein's equations controls the geometry) is uniformly distributed. We confine attention to spherical space, so that, strictly speaking, we should suppose the universe to be filled with matter of uniform density; but, practically, we need only insist on *large scale* uniformity, *i.e.* we suppose that the universe is filled with galaxies which are fairly regularly distributed everywhere.

If, further, it is postulated that this spherical universe is permanent and unchanging, there is but one unique solution, viz. Einstein's universe. For equilibrium, space must have a particular radius and contain a particular amount of mass (determinate in terms of the cosmical constant λ).

Technically, de Sitter's solution is also an equilibrium solution; but it is now realised that this is only because, being an entirely empty world, there is nothing in it to show departure from equilibrium.

2. *Expanding Universes.*—An infinite variety of solutions can be found representing spherical worlds which are not in equilibrium. Whilst remaining spherical they expand or contract, the radius (in

* *Annales de la Société Scientifique de Bruxelles*, **47**A, 49 (April 1927).

† *Nature of the Physical World*, chap. vii.

terms of our ordinary standards which are in a constant, though unknown, relation to the cosmical standard $1/\sqrt{\lambda}$) being a function of the time. In an expanding spherical world the galaxies, since they continue to fill space uniformly, must become further apart as time progresses. Expanding solutions are therefore of astronomical interest as a possible explanation of the observed scattering apart of the spiral nebulæ.*

As in Einstein's solution, the interval corresponding to spherical space and undistorted time is

$$ds^2 = -a^2\{d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)\} + dt^2 \quad . \quad (1)$$

where a is the radius of space and (χ, θ, ϕ) are angular co-ordinates †; but we now allow a to be a function of t. Lemaître shows that Einstein's gravitational equations give the following values for the density ρ and pressure p of matter in the space (1).

$$8\pi\rho = -\lambda + 3\left\{\frac{1}{a^2}\left(\frac{da}{dt}\right)^2 + \frac{1}{a^2}\right\} \quad . \quad . \quad . \quad (2)$$

$$8\pi p = \lambda - \left\{\frac{1}{a^2}\left(\frac{da}{dt}\right)^2 + \frac{1}{a^2}\right\} - \frac{2}{a}\frac{d^2a}{dt^2} \quad . \quad . \quad (3)$$

the units being such that the velocity of light and the constant of gravitation are unity. By (2) and (3)

$$\frac{6}{a}\frac{d^2a}{dt^2} = 2\lambda - 8\pi(\rho + 3p) \quad . \quad . \quad . \quad (4)$$

3. *Decay of Individual Motions.*—We shall regard a galaxy as being "at rest" if its angular co-ordinates (χ, θ, ϕ) remain constant. It is readily verified that its four-dimensional track will then be a geodesic. Observationally, galaxies "at rest" will appear to be receding from one another since the scale of the whole distribution is increasing. It is as though they were embedded in the surface of a rubber balloon which is being steadily inflated. Random individual motions relative to axes "at rest," whether of galaxies, of stars, or of atoms, constitute the pressure p in equation (3). By the usual kinetic theory p is $\frac{2}{3}\times$ kinetic energy per unit volume. To this should be added the pressure of radiation traversing space.

Lemaître obtains the striking result that when the universe expands the pressure changes *adiabatically*. The kinetic energy of the individual motions (analogous to heat) decreases by the amount of the work pdV done by the pressure in the expansion of volume of the universe. (The expansion, however, is not *caused* by the pressure; we shall find below that expansion occurs even when $p = 0$.) It follows that as the

* I confine attention to expanding universes for this reason; but the equations are always such that time is reversible, and to every expanding solution there corresponds a contracting solution.

† For distances small compared with a we can write $r/a = \chi = \sin\chi$ approximately; and (1) becomes the usual expression for spherical polar co-ordinates (r, θ, ϕ).

universe expands the individual motions of the galaxies tend to decrease, not only relatively to the scale of the system, but absolutely. In fact the average random velocity changes proportionately to $1/a$. Thus, if the expansion during past history has been considerable, we may expect the spiral nebulæ to be nearly "at rest," so that the regular scattering apart will not be unduly masked by individual motions.

4. *Conservation of Mass.*—It simplifies the tracing of the course of expansion of the universe if we may assume that the total mass of the universe remains constant. This is not rigorously true. We may consider either the proper mass or the relative mass (*i.e.* mass relative to axes "at rest"). Unfortunately neither of these is strictly conserved :—

(1) Apart from radiation the proper mass is conserved ; the relative mass diminishes owing to the decrease of kinetic energy of random motion mentioned in § 3.

(2) In the conversion of matter into radiation relative mass is conserved ; the proper mass diminishes since radiation has no proper mass.

Thus both masses may diminish a little in the course of time. However, these are rather insignificant complications. In what follows we shall generally take $p = 0$, so that proper mass and relative mass are the same, and both will be conserved to our order of approximation.*

5. *Instability of Einstein's Universe.*—Setting $p = 0$ in (4) we have

$$3\frac{d^2a}{dt^2} = a(\lambda - 4\pi\rho).$$

For equilibrium (Einstein's solution) we must accordingly have $\rho = \lambda/4\pi$. If now there is a slight disturbance so that $\rho < \lambda/4\pi$, d^2a/dt^2 is positive and the universe accordingly expands. The expansion will decrease the density ; the deficit thus becomes worse, and d^2a/dt^2 increases. Similarly if there is a slight excess of mass a contraction occurs which continually increases. Evidently Einstein's world is unstable.

The initial small disturbance can happen without supernatural interference. If we start with a uniformly diffused nebula which (by ordinary gravitational instability) gradually condenses into galaxies, the actual mass may not alter but the equivalent mass to be used in applying the equations for a strictly uniform distribution must be slightly altered. It seems quite possible that this evolutionary process started off the expansion of the universe. Once started, it must continue to expand at an increasing rate. I have not, however, been able to decide theoretically whether the condensation ought to start an expansion rather than a contraction.

Alternatively we might suppose that the initial equilibrium was upset by the conversion of matter into radiation. Such a conversion does not change ρ (since the mass of the radiation is equal to that of the matter converted), but it increases p. From equation (4) we see that

* Questions connected with the pressure are treated very fully in a forthcoming paper by Professor de Sitter.

the increase of p gives a negative d^2a/dt^2; thus the world would start contracting. This explanation must accordingly be rejected. It seems likely on general grounds that the conversion of matter into radiation would not begin until after a considerable degree of condensation into galaxies had occurred and the consequent expansion had got well under way.

A paper by R. C. Tolman * has just appeared suggesting the conversion of matter into radiation as an explanation of the recession of the nebulæ. It is clear, however, that it cannot be an "explanation," since by (4) it tends to retard the expansion. Tolman, in order to simplify the problem, imposes an artificial condition equivalent to $d(\log a)/dt = \text{const.}$ I think that what he determines in his paper is not a rate of conversion of matter which would "account for" the observed value of $d(\log a)/dt$, but a rate which would prevent $d(\log a)/dt$ from increasing.

We may compare the conditions of equilibrium (a) when the world contains only matter at rest, and (b) when it contains only radiation. Setting da/dt, $d^2a/dt^2 = 0$ in (2) and (3) we have

$$8\pi\rho = -\lambda + 3/a^2, \qquad 8\pi p = \lambda - 1/a^2,$$

so that

$$1/a^2 = 4\pi(\rho + p), \qquad \lambda = 4\pi(\rho + 3p).$$

(a) For matter at rest, $p = 0$. Hence

$$a^2 = 1/\lambda.$$

(b) For radiation, $\rho = 3p$. Hence

$$a^2 = \tfrac{3}{2}/\lambda.$$

The total masses are found to be

$$(a)\ \frac{\pi}{2}\lambda^{-\frac{1}{2}}, \qquad (b)\ \frac{\pi}{4}\left(\frac{3}{2}\right)^{\frac{3}{2}}\lambda^{-\frac{1}{2}}.$$

The difference of mass in these two extreme cases is only 8 per cent.

6. *Rate of Expansion.*—Taking $p = 0$, let

a_e, M_e be the radius and mass of an Einstein universe,
a, M the radius and mass of the system under consideration.

We recall that a is a function of the time and the other three quantities are constants. Results for Einstein's universe are

$$\frac{2}{\pi}\mathrm{M}_e = a_e = 1/\sqrt{\lambda}\ . \qquad . \qquad . \qquad . \qquad (5)$$

The whole volume of the spherical world is $2\pi^2 a_e{}^3$; hence the density $\rho_e = 1/4\pi a_e{}^2 = \lambda/4\pi$, as already found. The mass is here measured

* *Proc. Nat. Acad. Sci.*, **16**, 320 (April 1930). Lemaître in his concluding paragraph seems also to entertain the same view.

in the gravitational unit, which is such that the sun's mass is approximately 1·5 kilometres. By (2)

$$\left(\frac{da}{dt}\right)^2 + 1 = \tfrac{1}{3}a^2(\lambda + 8\pi\rho)$$
$$= \tfrac{1}{3}a^2\lambda + 4\mathrm{M}/3\pi a.$$

Hence

$$\frac{da}{dt} = \sqrt{(\tfrac{1}{3}a^2\lambda - 1 + 4\mathrm{M}/3\pi a)} \quad . \quad . \quad . \quad (6)$$

Three cases arise:—

(*a*) If $\mathrm{M} > \mathrm{M}_e$, the right-hand side does not vanish for any positive value of a, and the system can expand continuously from very small to very large radius. The minimum of da/dt is given by differentiating (6) with respect to a,

$$\tfrac{2}{3}a\lambda - 4\mathrm{M}/3\pi a^2 = 0,$$

so that $a^3 = 2\mathrm{M}/\pi\lambda$, or using (5)

$$a/a_e = \sqrt[3]{(\mathrm{M}/\mathrm{M}_e)}.$$

As the radius increases through this value of a, the rate of expansion slows down and increases again. The difficulty of applying this case is that it seems to require a sudden and peculiar beginning of things.

(*b*) If $\mathrm{M} < \mathrm{M}_e$, the right-hand side vanishes for two positive values of a, say R_1, R_2, and is imaginary for values of a, intermediate between R_1 and R_2. Hence either the world starting with a finite velocity of expansion expands to radius R_1 and then contracts again, or starting with a finite velocity of contraction contracts to radius R_2 and increases again. It is difficult to find a natural starting-point for the actual universe on the locus mapped out for it. If we decide to accept case (*b*) we should presumably assume that the actual universe began with radius R_2, so that initially $da/dt = 0$; since then it has continually expanded.

As $a \to \mathrm{R}_2$, $da/dt \to 0$ like $\sqrt{(a - R_2)}$, and it follows that the radius remains in the neighbourhood of R_2 for a finite time only. With any reasonable estimate of the present degree of expansion, the date of the beginning of the universe is uncomfortably recent.

(*c*) The limiting case is when $\mathrm{M} = \mathrm{M}_e$ as in § 5. Then R_1 and R_2 coalesce to the value a_e. As $a \to a_e$, $da/dt \to 0$ like $(a - a_e)$; hence the time that the radius remains in the neighbourhood of a_e is logarithmically infinite. There is at least a philosophical satisfaction in regarding the world as beginning to evolve infinitely slowly from a primitive uniform distribution in unstable equilibrium. Hence this case is the most attractive. But in physics logarithmic infinities are usually deceptive, and when adapted to practical applications turn out to be not so very large. I do not think we prolong the effective age of the galaxies very much by accepting $\mathrm{M} = \mathrm{M}_e$ rather than case (*b*). We allow evolution an infinite time to get started; but once seriously started its time-scale of progress is not greatly different from case (*b*).

When $M = M_e$, (6) can be integrated without elliptic functions. The result, which is rather complicated, will be found in Lemaître's paper (equation (30)).

7. *Seeing Round the World.*—A question, debated perhaps more for its picturesqueness than for its practical importance, is whether we can see right round the universe. It is well known that this is possible in Einstein's world, but not in de Sitter's world. Lemaître's expanding universe is of intermediate type, and curiously the answer also is rather intermediate: we can see all parts of the world, but it is not likely that we shall be seen from all parts of the world.

A light-track is given by setting $ds = 0$ in (1). Taking ourselves to be at the origin $\chi = 0$ at time t_0, we are on a light-track given by

$$0 = -a^2 d\chi^2 + dt^2,$$

so that for the past (events at χ, t seen by us now)

$$\chi = \int_t^{t_0} \frac{dt}{a} \quad . \quad . \quad . \quad . \quad (7)$$

and for the future (an observer at χ, t who sees us as we are now)

$$\chi = \int_{t_0}^{t} \frac{dt}{a} \quad . \quad . \quad . \quad . \quad (8)$$

Since in the past a was always less than at present, χ in (7) increases indefinitely as $t \to -\infty$; so that we can see many times round the world, if the world has endured long enough. In the future a will continually increase at an increasing rate, and it is possible that the integral (8) may tend to a finite limit less than 2π as $t \to \infty$. Lemaître, using estimates from astronomical data, calculates that this actually is the case. Observers whose co-ordinate χ is beyond this limiting value can never see the events now happening in our galaxy; they see the earlier events, but their infinite lifetime only suffices to follow our history up to a certain moment at which the rate of expansion of the radius of the world became too rapid.

To put it another way—the velocity of light in angle (*i.e.* fraction of the whole circuit round spherical space) decreases as the radius of space increases. At first when the radius is small a light-wave can make a number of complete circuits, but ultimately it can only approach asymptotically a fixed position.

Out of curiosity I have calculated by Lemaître's formulæ the time when circumambulation by light ceased to be possible. This happened very early in the expansion when a had only reached the value $1{\cdot}003a_e$. (This applies to case (*c*).) When the expansion reached this amount every light-wave then existing entered on its last lap; waves that started later can never complete a circuit. I may add that in so-called "elliptical space" a short cut is provided which halves the distance to be traversed; by cheating in this way the light can return to its starting-point if it starts not later than the moment when $a = 1{\cdot}073a_e$.

8. *Spectral Shift.*—Taking ourselves to be "at rest" at $\chi = 0$

and an object seen by us to be " at rest " at some constant χ, we obtain by varying the limits of the integral in (7),

$$0 = \frac{\delta t_0}{a(t_0)} - \frac{\delta t}{a(t)}$$

where $a(t)$ is the radius of the world at time t. Hence

$$\frac{\delta t_0}{\delta t} = \frac{a(t_0)}{a(t)} \quad . \quad . \quad . \quad . \quad (9)$$

that is to say, the ratio of the period observed to the period emitted is equal to the ratio of the radius of the universe at the time of observation to the radius at the time of emission. This determines the amount of red-shift of the spectrum.

We can immediately find the present rate of expansion of the universe. The average red-shift of the spectra of spiral nebulæ amounts to about 500 km. per sec. per megaparsec, or about $\frac{1}{2000}$ of the velocity of light for a million light-years distance. Hence we set $\delta t_0/\delta t = 2001/2000$ in (9) for an interval $t_0 - t = 1{,}000{,}000$ years. Hence the radius of the universe has expanded by 1 part in 2000 in the last million years.

The result is impressive. It indicates that the radius of space has doubled within ordinary geological time. Let

$$\frac{1}{a}\frac{da}{dt} = \frac{1}{\mathrm{A}}.$$

Then the foregoing result gives

$$\mathrm{A} = 2{\cdot}10^9 \text{ light-years.}$$

This gives a hint of the scale of the phenomenon that we are concerned with. By (5) we can calculate that a (fictitious) Einstein world * of radius A would have a mass $2 \times 10^{22} \odot$ and a density

$$\rho_{\mathrm{A}} = 3{\cdot}10^{-28} \text{ gm./cm.}^3$$

Remembering that the density in the neighbourhood of the sun is of order 10^{-24} or at the most 10^{-23}, and allowing for large comparatively empty regions between the galaxies, it is probable that the density ρ of the actual universe is considerably less than ρ_{A}. This conclusion will be of use in § 9.

If we attempt to look a long way back into the past, as in the problem of " seeing round the world ", the ratio $a(t_0)/a(t)$ will probably be large, and what was originally visual light may be shifted into the infra-red. There is thus in an expanding universe a kind of gradual obliteration of the past through fading into longer and longer wave-lengths. There

* There is only one possible radius for an Einstein world (with $p = 0$) which is at present unknown. We are here calculating (for purposes of comparison) what would be the density if the unknown radius turned out to be A. We shall find later that it is actually less than A.

is a well-known speculation concerning "ghosts" of stars or nebulæ formed by reconvergence of the emitted light after passing round the world. (This is fanciful because the actual irregularity of the universe would be almost sure to spoil the convergence.) These ghosts become redder with age, and disappear out of our visual range.

9. *Numerical Estimates.*—Even if we assume for the theoretical reason referred to in § 6 that $M = M_e$, a further astronomical datum besides A is necessary in order to determine a. The required datum is an estimate of the actual mean density of the world. At present it is not possible to make an estimate that would be of any serious value.

Let $a/a_e = q$, so that q is a measure of the expansion that is supposed to have occurred since the original equilibrium condition collapsed. For $M = M_e$ we obtain from (5) and (6)

$$\frac{da}{dt} = \sqrt{(\tfrac{1}{3}q^2 - 1 + 2/3q)},$$

so that

$$\frac{1}{A^2} = \frac{1}{a^2}\left(\frac{1}{3}q^2 - 1 + \frac{2}{3q}\right) \quad . \quad . \quad . \quad (10)$$

Also

$$4\pi\rho = \frac{2M_e}{\pi a^3} = \frac{1}{a^2 q}.$$

Hence

$$\frac{\rho_\Lambda}{\rho} = \frac{1}{4\pi\rho A^2} = \frac{1}{3}q^3 - q + \frac{2}{3} \quad . \quad . \quad . \quad (11)$$

For our expanding universe $q > 1$, and it follows from (11) that $\rho_\Lambda > \rho$. It is satisfactory that this accords with our observational conclusion in the last section. If we are right in believing that ρ is *considerably less* than ρ_Λ, q will be large and (11) and (10) become approximately

$$\frac{\rho_\Lambda}{\rho} = \frac{1}{3}q^3; \qquad \frac{a^2}{A^2} = \frac{1}{3}q^2 \quad . \quad . \quad . \quad (12)$$

Hence

$$a_e^2 = \tfrac{1}{3}A^2 \quad . \quad . \quad . \quad . \quad (13)$$

so that

$$a_e = 1200 \text{ million light-years.}$$

Estimates of ρ, such as that made by Hubble, may perhaps be expected to be valid to within a factor 100. The corresponding determination of q and a is uncertain to a factor $(100)^{\frac{1}{3}}$. Thus our knowledge of the present radius of space remains very vague.

We conclude that the radius of space was originally about 1200 million light-years, that it has since expanded considerably, but to an amount practically undeterminable, and that its present rate of expansion is 1 per cent. in about 20 million years—a rate which will continue indefinitely.*

* It is seen from (6) that $\frac{1}{a}\frac{da}{dt}$ tends to a fixed limit $\sqrt{\frac{1}{3}\lambda}$ as $a \to \infty$.

10. *Miscellaneous Questions.*—De Sitter's world corresponds to $\rho = 0$, and is the limit approached as the density diminishes through expansion of volume of space. Accepting case (*c*), $M = M_e$, the history of expansion of the universe resolves itself into a gradual transition from Einstein's to de Sitter's world. According to § 9 the expansion has now proceeded so far that the de Sitter model gives much the better approximation; but this depends entirely on the truth of our estimate that the actual mean density of the universe is much less than 10^{-28} gm./cm.^{3} Admitting this, no great change is required in current theories which have assumed a de Sitter universe, provided that they do not concern themselves with early history.

De Sitter, however, used a different reckoning of space and time and "at rest" from that used in (1), so that the same phenomena were described by him in a different way. In particular he introduced "the slowing down of time" at great distances from the origin, which does not occur in the new formulæ. The present description involves fewer paradoxes and is undoubtedly easier to apprehend—as several investigators had already discovered. It has, moreover, the advantage that we now approach de Sitter's world as the limit of a series of worlds of gradually diminishing density; whereas formerly we had to start with a completely empty world, and very cautiously put a few material bodies into it.

It was originally urged against Einstein's world that it restored to a certain extent the absoluteness of space and time. The same applies to Lemaître's expanding universe in which there is a natural definition of "at rest." There is, however, no real conflict with relativity. The existence of a unique standard of reckoning is due to the artificial condition that we have imposed, viz. the complete homogeneity of ρ and p; for a universe which is not perfectly spherical the standard no longer exists. The breakdown can be illustrated as follows. In a perfectly spherical Einstein universe the rays of light from a star will, after going right round the world, converge to form a real image or ghost-star at a point P, the star itself having meanwhile moved to a point P′. In general P′ will not coincide with P; in fact after emitting the light the star may have burst into fragments which have scattered to a number of different points P′. We regard PP′ as an absolute motion of the star, and accordingly define a body "at rest" as one which coincides with its own ghost. If the world is not perfectly spherical the focussing of the reconverging rays will be imperfect, and the image will be diffused over a large area. The ghost no longer defines a definite point, and our definition of "at rest" fails. This is entirely analogous to absolute rotation; in a perfectly uniform world a quantity can be defined which with some justification may be called "absolute rotation"; in an uneven world the definition fails, but the deviation from Euclidean geometry in the actual universe is so slight that absolute rotation can be retained as an approximate conception.

For general qualitative reasoning it is useful to remember that the λ term in the gravitational equations is equivalent to a repulsive force from the origin varying directly as the distance, and the phenomena

depend essentially on whether this repulsion does or does not prevail over the ordinary gravitational attraction of the matter present. Except in a closed space the repulsion must prevail at sufficiently great distances. It is only when we have to deal with a region so large that its total curvature amounts to a considerable fraction of a sphere that this simple outlook begins to fail, mainly because new questions arise as to the conventional definitions of distance, simultaneity, etc., when the region can no longer be treated as approximately flat.

11. *The Time-scale.*—The rapid expansion of the universe now proceeding is evidently unfavourable to a long time-scale of billions of years. It is difficult to produce a decisive objection, but there is a general incongruity. We cannot calculate how long a period elapsed from the disturbance of Einstein equilibrium by the beginnings of evolutionary development until the deviation from equilibrium reached a serious amount; but from the time when the universe had reached, say, 1·5 times its initial radius to the present day, it is scarcely possible to allow more than 10^{10} years. If the sun has really existed as a star for 5 billion years, it is odd that it should have waited so long and then formed its system of planets just at the time the universe toppled into a state of dispersion.

The universe is now doubling its radius every 1400 million years, and this rate will, if anything, slightly increase in the future. In 10^{10} years the spiral nebulæ will be 10 magnitudes fainter than they are now. With a time-scale of billions of years, astronomers must count themselves extraordinarily fortunate that they are just in time to observe this interesting but evanescent feature of the sky.

This last conclusion is not dependent on the theory of spherical space; it rests on the present observational evidence that (apart from three or four near nebulæ) the spirals are receding at a rate which must take them outside our range of observation in a comparatively short time, and that none are coming in to replace them. Unless a theory is invented which provides some force opposing this recession, there is no evading the rapid departure of nebulæ from our neighbourhood.

12. *Conclusion.*—The proof of the instability of Einstein's model greatly strengthens our grounds for interpreting the recession of the spiral nebulæ as an indication of world curvature. When this explanation was first suggested by de Sitter, all that was known was that of the two models known to satisfy Einstein's accurate equations one would exhibit a phenomenon of this kind. Now the position is that in any world that satisfies Einstein's equations, such a phenomenon (or its opposite, *i.e.* a general velocity of approach) must necessarily appear in the course of time; the phenomenon is not merely consistent with theory, but is foretold by theory. Of course it is possible that the recession of the spirals is not the expansion theoretically predicted; it might be some local peculiarity masking a much smaller genuine expansion; but the temptation to identify the observed and the predicted expansions is very strong.

The investigation is incomplete in that we have only been able to study a system of galaxies strewn all over the world. It would be

desirable to supplement this by considering cases in which the material system is confined to a part of space; the problem presents greater mathematical difficulties, but these are perhaps not insuperable.

The theory gives a value of the total mass of the universe which ought to be fairly trustworthy. Since the mass initially formed an Einstein universe of radius 1200 million light-years (§ 9), it is found by the usual formulæ to be

$$1{\cdot}1 \,.\, 10^{22} \odot \;=\; 2{\cdot}3 \,.\, 10^{55} \text{ gm.}$$

It may be recalled that the only astronomical datum used in obtaining this value is the red-shift of the spectra of the nebulæ estimated at 500 km. per sec. per megaparsec. It is also postulated (1) that $M = M_e$, since this condition is necessary if the universe is to have a natural beginning in accordance with general evolutionary ideas; (2) that the mean density of matter throughout the universe is considerably less than 10^{-28}.

It may be noted that this gives a total of $1{\cdot}4 \,.\, 10^{79}$ protons in the universe. Another way of reckoning gives half this number, as it is possible that one-half of the spherical world is merely a mathematical duplication of the other half.

The Masses, Luminosities, and Effective Temperatures of the Stars.
Second Paper. By E. A. Milne, F.R.S.

1. Though I have studied with great care Professor Eddington's two papers in *Monthly Notices*, 1930 January, I can find nothing in them which requires me to withdraw or modify any of my paper on "The Masses, Luminosities, and Effective Temperatures of the Stars" in *Monthly Notices*, 1929 November. Professor Eddington has misunderstood the range of ideas developed in my paper, and he nowhere makes real contact with my arguments. As I stated at the beginning of my paper, my approach to the subject is totally different from Professor Eddington's, but he completely ignores the consequences of the changed point of view.

In his first paper Eddington replies to an accusation I did not make, and in his second paper he makes a serious mistake in reversing some of my algebra, and so overlooks my main theorem.

Professor Eddington says: "he (Milne) believes that the photospheric opacity is a leading factor in determining the luminosity." I neither said nor implied any such thing. The luminosity of a star in a steady state is simply equal, and always equal, to the total source strength, whatever the surface opacity, or for the matter of that, the interior opacity. I said and proved that the rate of cooling of a star of given mass and *given relative density-distribution** depends on its surface opacity. To assume a steady state it must adjust either its

* *I.e.* belonging to a given homologous family.

relative density-distribution or its surface opacity or both, so that the total rate of generation of energy (prescribed beforehand) is equal to the rate of cooling. In particular, *if* it retains a constant relative density-distribution it must adjust its surface opacity, which it can do by altering its radius and so its photospheric temperature. I left it an entirely open question which of the two ways of adjustment we should expect a star to choose. I simply examined the stars occurring in nature to see which method of adjustment they have in fact taken. The answer to Eddington's question : " What is the resulting change of luminosity when the opacity of the surface layers is altered ? " is immediate from the standpoint I adopted. The change of luminosity is rigorously zero. The total generation of energy, prescribed beforehand, is unaltered, and in the steady state this is equal to the luminosity.

In his second paper Professor Eddington has not noticed that my work contains as a particular case the general solution of the problem of the gaseous star with β and μ variable through the star, under precisely the same boundary conditions as for his model, in which β and μ are constant ; and my solution reduces to his when β and μ are constant. I display the complete parallelism in § 3 below. Thus Eddington's objections to my solution, if valid, would be equally fatal to his own solution for β and μ constant. Actually his criticism is based on an oversight ; he integrates my equation (24), in which θ has already been given the particular value zero, as though it held for all values of θ. When we restore the term which disappeared when θ was put equal to zero, we merely recover the equation of mechanical equilibrium, as shown by Sir Joseph Larmor.* I verify this below, § 5.

I claim no originality for making use of the principle of the avoidance of unnecessary hypotheses. That principle, I need hardly remind Professor Eddington, goes back at least as far as William of Occam. The only way we can test whether results verify a hypothesis is to conduct the same investigation as far as possible without the hypothesis, or with alternative hypotheses, and again compare with observation. I nowhere criticised the plausibility of Professor Eddington's *physical* hypotheses about stellar interiors ; I merely showed that the perfect-gas hypothesis was not a necessary one. When I used the word " arbitrary," Professor Eddington must surely have realised that I was referring to his mathematical hypothesis " $\kappa\eta$ = constant " or its equivalent " β = constant." Equation (30) of my paper shows that when the star is composed of perfect gas, my function ϕ is given by

$$\phi^3 = \frac{1-\beta}{\beta^4\mu^4} \cdot \frac{\beta_c^4\mu_c^4}{1-\beta_c} \quad . \quad . \quad . \quad . \quad (1)$$

and thus my solution for any ϕ (in terms of a differential equation that can be solved by quadrature) includes as a particular case the solution for the gaseous star with arbitrary march of β and μ from centre to boundary. The removal of the arbitrary assumptions, " β = constant " and " μ = constant," *i.e.* the removal of the assumption $\phi = 1$,

* *Observatory*, **53**, 115, 1930.

discloses the existence of a boundary condition, for prescribed M and L. The boundary condition imposes a restriction on the possible forms of ϕ, or, what is the same thing, on the relative density-distribution. This restriction is simply that the relative density-distribution must be such that the boundary layers simultaneously (*a*) enclose the prescribed mass M, (*b*) radiate to space at the prescribed rate L.

Professor Eddington's problem is: given the mass M and the relative source-distribution, what is the radiation L to space in the steady state? As explained in the introduction to my paper, and for the reasons there stated,* the problem I discuss is: given the mass M and the total generation of energy L, what is the source-distribution in the steady state? In the steady state, the radiation to space (the luminosity) is equal to the prescribed generation L. I discuss this problem by first ascertaining what density-distributions (specified by ϕ) are compatible with the prescribed M and L. I find that the only restriction imposed on ϕ is that a certain coefficient C associated with ϕ (and depending on the complete march of ϕ from centre to boundary) must have a certain numerical value. This number depends on the prescribed M and L and on the opacity of the boundary layers. If this opacity is prescribed absolutely, C can be determined immediately; if merely the opacity-law is prescribed, we require to know the photospheric temperature, *i.e.* we require a prescribed T_1. Thus, for any observed star, C can be found, and hence the restriction on its ϕ, *i.e.* on its relative density-distribution. Actual functions ϕ satisfying the restriction thus derived could be found by trial and error, with the aid of numerical quadratures of the differential equation defining C; such quadratures are now being carried out by my pupil Mr. Norman Fairclough, M.Sc., though they are very laborious.

It is clear that the ensemble of solutions obtained by Professor Eddington's method of attack must coincide with the ensemble obtained by mine. Hence it is desirable that I should state exactly where my specific results differ from his.

In the first place our solutions are identical for the density-distribution for which β is constant. When β is not constant, Professor Eddington gives the formula

$$L = \frac{4\pi cGM(1-\overline{\beta})}{\overline{\kappa\eta}} \quad . \quad . \quad . \quad . \quad (2)$$

Assuming Eddington is right in his conjecture that when β is not constant his quartic equation (with coefficient C calculated as though β were constant) *gives a mean value* $\overline{\beta}$, equation (2) is valid. But the weighting of $\overline{\kappa\eta}$ is not specified. Equation (2) is not affected by the existence of a boundary condition—it is too vague.† It is as though we were calculating the volume of an ellipsoid from the formula

* And for other reasons stated in *Nature*, 1930 March 22.

† The vagueness of this formula is inherent in its derivation. Eddington attempts to deduce L/M by discussing the mean value of $L(r)/M(r)$ between centre and boundary. Clearly what we want is its limit as r tends to the radius of the star.

$V = \frac{4}{3}\pi\bar{r}^3$, in ignorance of the exact formula $V = \frac{4}{3}\pi abc$. The existence of a restriction, for example, on the ratios $a : b : c$ does not affect the validity of the mean-value formula. As soon as the exact formula is known, the mean value formula becomes obsolete, though it conveys information about the order of magnitude of $\bar{r}$. Similarly, in view of my exact formulæ, (2) becomes obsolete, though for prescribed M and L it conveys information about the order of magnitude of $\overline{\kappa\eta}$. If κ in the interior is prescribed, it conveys information about the order of magnitude of $\bar{\eta}$. The formulæ deduced by Eddington from (2) by assigning a particular value to $\bar{\eta}$ are not valid, for as far as can be ascertained they are incompatible with the prescribed L and M of stars occurring in nature, as shown by Eddington.*

When we have found a density-distribution compatible with the prescribed L, M, and κ_1 (boundary opacity) we can assign any *arbitrary* distribution of κ to the interior (tending to κ_1 at the boundary) and determine the source-distribution by differentiation, as shown in my paper. If it is desired to reject all distributions involving sinks of energy, further conditions will be imposed, but we see that in general a knowledge of L and M only is not sufficient to determine κ. If the actual sources were known, *i.e.* if the laws of generation of sub-atomic energy were known, we could determine κ in the interior as that distribution of opacity in the interior which gave the required sources. Thus, ultimately we can look forward to determining κ for the interior, but not in the present state of knowledge. In the meantime κ for the interior is not ascertainable. This removes the discrepancy between the theoretical and the supposed " astronomical " value of κ.

The mass-luminosity correlation observed in nature must convey real information about the relation of the energy-generating process to the mass of the star. On Professor Eddington's view it is a consequence of steady-state considerations only. On the present view the stars occurring in nature are a sub-class of the ensemble of possible steady-states corresponding to arbitrary pairs (L, M).

I proceed to discuss in detail some of the points raised by Professor Eddington.

2. If we change the surface opacity of a star, the answer to the question " How is the luminosity changed ? " is, for the problem that I discuss, that there is no change in the luminosity, because in the steady-state the luminosity is equal to the rate of generation of energy, which by hypothesis is unchanged. The rate of radiation from the surface is given by

$$\frac{4\pi c\mathrm{GM}(1-\beta_1)}{\kappa_1} \quad . \quad . \quad . \quad . \quad (3)$$

where $1 - \beta_1$ depends on the mass, photospheric molecular weight, and complete relative density-distribution. For this expression to be

* Eddington's use of a particular value for $\bar{\eta}$ rests to a large extent on his particular point-source solution, but on physical grounds there must exist other point-source solutions of smaller luminosity and the same mass, as I have shown. Thus $\bar{\eta}$ can take arbitrarily large values.

conserved on change of κ_1 (or a constant occurring in the physical law for κ_1), either the effective temperature must be changed to such a value that the old value of κ_1 is restored; or the relative density-distribution must change, thus changing $1 - \beta_1$; or both effects must occur. But if the total generation remains unaltered we can be quite sure that when the steady-state is reached the above expression remains unaltered. The numerical value of C is determined as before, from the changed surface opacity. There are an infinite number of density-distributions compatible with the old L and M and the new surface opacity; and as I stated I have not found a condition which uniquely determines which density-distribution will be adopted. The new C may or may not be equal to the old.

Professor Eddington in his solution keeps the relative source-distribution unchanged, though the absolute source-intensity integrated through the star is slightly altered. Professor Eddington gives no physical reason why the sources should insist on conserving their relative distribution but be prepared to radiate at a new rate dictated by the surface opacity. His solution implies, in fact, some specific interaction between surface opacity and source-strength. This is a real physical difficulty.

Further consideration shows that he has not really solved his own problem. He first changes the mass (by 1 part in 10^{10}), keeping L/M the same. Then, at the end of his paper, he mentions "an effect on L" due to "changes of radius," which he does not calculate. So that, after all, L/M is not kept the same. In what way can the change of radius affect the luminosity (which in the steady-state is always equal to the total generation) without some specific reaction of temperature on source-intensity which we know nothing about?

As Professor Eddington accuses me of introducing confusion into the subject comparable with what would be caused by two rival theories of quadratic equations, I feel compelled to draw attention to a mistake in differential-equation theory made by him in his first paper. The analysis of his equations (1) to (6) on p. 281, is not new. It was given by me in identical terms in *Monthly Notices*, **85**, 784, 1925, his equations (4), (5), (6) being my equations (34), (35), (36). There I correctly attributed the origin of this type of analysis to earlier work by Jeans and Woltjer, and last year * I repeated the analysis, taking account of an extra term $T^{-\frac{1}{2}}$ in κ. On all these occasions Eddington's equation (4) was solved with the correct boundary condition for radiative equilibrium for any photospheric opacity κ_1 (independent of interior opacity κ). Eddington, however, remarks, "When $\kappa_1 = \kappa$, the well-known solution is $z(= p_R/P) =$ constant . . . so that $z = \alpha$." This is not true. $z = \alpha$ is only the singular solution of Eddington's (4), *i.e.* a solution not included in the complete primitive. It only holds when p_R and p vanish simultaneously at the boundary.† This is a

* *M.N.*, **90**, 33, 1929, equations (43), (44), (45); *Phil. Trans.*, **228A**, 456, 1929, equations (75), (76), (77).

† If p and p_R are taken to vanish simultaneously at the boundary in the case κ, they should also be taken to vanish simultaneously at the boundary in the case κ_1.

sufficient approximation when the whole interior is being investigated, but is not suitable in a specific investigation of the photospheric layers, for actually p_R remains non-zero when p vanishes. It is easily found, and has been long known, that in this case the value of z approaches α asymptotically and rapidly. This is a very important point in the theory of the photospheric layers.

But when we change κ to κ_1, Eddington's σ changes to a new value σ_1 (if L/M is kept constant), α changes to a new value α_1, and z asymptotically approaches α_1, *not* α. Eddington says that z is equal to $\alpha + \epsilon$, where ϵ rapidly diminishes. This is false. He omits to notice that when he changes his κ to κ_1, his σ is changed, and so α and α'. Before putting $z = \alpha + \epsilon$ in (6) he should have replaced α and α' in (6) by α_1 and α_1'. This vitiates his analysis as a discussion of the equilibrium of the outer layers of a star.

3. I now propose to repeat some of the analysis of my paper to show that my solution for variable β and μ is on precisely the same footing as Eddington's solution for constant β and μ, for a star composed of perfect gas.

The equations of the problem are

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho \quad . \quad . \quad . \quad . \quad (4)$$

$$\frac{dp'}{dr} = -\frac{\kappa\rho F_r}{c} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\epsilon_r = \frac{1}{4\pi r^2\rho}\frac{d}{dr}(4\pi r^2 F_r) \quad . \quad . \quad . \quad (6)$$

Here P is the total pressure, p' the radiation-pressure, F_r the net flux of radiant energy per cm.^{2} across the sphere of radius r, ϵ_r the rate of generation per unit mass. Equation (4) is the condition of mechanical equilibrium. Any distribution of ρ and T satisfying (4) is such that the acceleration of each element is instantaneously zero. Equation (5) is the equation of transfer. It tells us what is the net flux of radiant energy at any point caused by the particular temperature-distribution in question. It holds whether or no the star is in a steady thermal state.* Equation (6) determines the sources required to maintain the star in a steady-state thermally. Given any distribution of ρ and T, together with an equation of state permitting the determination of gas-pressure p from ρ and T, we can derive F_r from (5), provided κ is known, and then from (6) we can find the corresponding sources necessary to ensure a steady-state. The problem is (Eddington's formulation) to obtain the surface flux $L = (4\pi r^2 F_r)_{r=r_1}$ for given total mass M, given the *relative* source-distribution ϵ_r; or (my formulation) to determine the density-distribution compatible with a given M and L,

* If the star is not in a steady thermal state, (5) requires a correcting term in the neighbourhood of the boundary (see *Phil. Trans.*, **228**A, 430, equation (291), but this correcting term vanishes approximately in radiative equilibrium (*loc. cit.*, p. 453), so that in the thermally steady-state (5) holds right up to the boundary.

and then determine the corresponding source-distribution, which must necessarily total to L.

Now assume the material is perfect gas of molecular weight μ, and write

$$p = (\mathrm{R}/\mu)\rho\mathrm{T} = \beta\mathrm{P},$$

$$p' = \tfrac{1}{3}a\mathrm{T}^4 = (1-\beta)\mathrm{P}.$$

Then

$$\frac{\mathrm{P}^3}{\rho^4} = \frac{1-\beta}{\frac{1}{3}a\beta^4}\left(\frac{\mathrm{R}}{\mu}\right)^4 . \quad . \quad . \quad . \quad (7)$$

The ideal model for which we propose to solve the equations is that in which ρ and T vanish together at the boundary. Whether this suitably represents an actual star, in which T is non-zero when ρ vanishes, is a matter for separate discussion. I gave a full discussion of the point in my paper.* The result of that discussion was (and it is a result always assumed by Eddington) that the model in which ρ and T vanish simultaneously at the boundary has the same radiation to space (apart from a small correction there calculated) as the model in which T is non-zero at the vanishing of ρ. It is the boundary condition of all Eddington's and Emden's models.†

Having decided on our model we stick to it rigorously. We now consider the sub-class of such models in which β and $1-\beta$ are always non-zero.‡ Then p/p' is finite at the boundary, *i.e.* ρ vanishes like T^3 at the boundary, and P^3/ρ^4 is non-zero at the boundary.

From (7), if suffix c denotes central values,

$$\frac{\mathrm{P}^3}{\rho^4} = \frac{\mathrm{P}_c^3}{\rho_c^4}\frac{1-\beta}{\beta^4\mu^4}\frac{\beta_c^4\mu_c^4}{1-\beta_c} = \phi^3\frac{\mathrm{P}_c^3}{\rho_c^4}, \quad . \quad . \quad . \quad (8)$$

say, where

$$\phi = \left[\frac{1-\beta}{\beta^4\mu^4}\cdot\frac{\beta_c^4\mu_c^4}{1-\beta_c}\right]^{\frac{1}{3}} \quad . \quad . \quad . \quad (9)$$

Then for the model considered ϕ is everywhere finite and non-zero. Differentiating the equation of mechanical equilibrium (4), we find

$$\frac{1}{r^2}\frac{d}{dr}\left[\frac{r^2}{\rho}\frac{d\mathrm{P}}{dr}\right] = -4\pi\mathrm{G}\rho \quad . \quad . \quad . \quad (10)$$

or inserting for P from (8)

$$\frac{\mathrm{P}_c}{4\pi\mathrm{G}\rho_c^2}\frac{1}{r^2}\frac{d}{dr}\left[\frac{r^2}{\rho/\rho_c}\frac{d}{dr}\left\{\phi\left(\frac{\rho}{\rho_c}\right)^{\frac{4}{3}}\right\}\right] = -\frac{\rho}{\rho_c} \quad . \quad (11)$$

We now introduce for convenience Emden's variable θ, defined by

$$\rho = \rho_c\theta^3, \quad . \quad . \quad . \quad . \quad (12)$$

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* *Monthly Notices*, **90**, 32, 1929, section 20.

† I have actually carried through (but not published) the theory when $\mathrm{T}\to\mathrm{T}_0$ as $\rho\to 0$, and though it is more complicated no essential change results.

‡ If $1-\beta$ vanishes at the boundary, the radiation to space comes out zero and there is a distribution of sinks in the outer portions.

and put

$$\frac{P_c}{4\pi G\rho_c^2}\frac{1}{r^2} = \frac{1}{x^2} \quad . \quad . \quad . \quad . \quad (13)$$

Then (11) becomes

$$\frac{1}{x^2}\frac{d}{dx}\left[\frac{x^2}{\theta^3}\frac{d}{dx}\left\{\phi\theta^4\right\}\right] = -\theta^3 \quad . \quad . \quad (14)$$

(*a*) *Eddington's solution for* β *and* μ *constant.*—So far our work has involved no assumptions about the behaviour of β and μ except that β is finite and non-zero as $\theta \to 0$. Now adopt Eddington's assumptions, β = constant and μ = constant. Then ϕ is constant and equal to unity. Equation (14) becomes

$$\frac{4}{x^2}\frac{d}{dx}\left(x^2\frac{d\theta}{dx}\right) = -\theta^3 \quad . \quad . \quad . \quad . \quad (15)$$

or, putting $x = 2\xi$,

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^3,$$

which is Emden's differential equation for the polytrope $n = 3$. This has to be solved with the conditions that at $\xi = 0$, $\theta = 1$, and $d\theta/d\xi = 0$. The solution is continued numerically until θ vanishes, which occurs, say, at $\xi = \xi_1$, or $x = x_1 = 2\xi_1$. From the numerical solution we know the value of $(d\theta/dx)$ at $x = x_1$, say $(d\theta/dx)_1$.

We now find the total mass. We have

$$\begin{aligned} M(r) = \int_0^r 4\pi\rho r^2 dr &= 4\pi\rho_c\left(\frac{P_c}{4\pi G\rho_c^2}\right)^{\frac{3}{2}}\int_0^x \theta^3 x^2 dx \\ &= -\frac{4^{\frac{1}{2}}}{\pi^{\frac{1}{2}}G^{\frac{3}{2}}}\frac{P_c^{\frac{3}{2}}}{\rho_c^2}\left(x^2\frac{d\theta}{dx}\right) \quad . \quad . \quad (16) \end{aligned}$$

But by (7), β and μ being constant, the value of the constant P_c^3/ρ_c^4 is

$$\frac{P_c^3}{\rho_c^4} = \frac{1-\beta}{\frac{1}{3}a\beta^4}\left(\frac{R}{\mu}\right)^4 \quad . \quad . \quad . \quad . \quad (17)$$

Hence

$$\left[M(r)\right]^2 = \frac{4}{\pi G^3}\frac{1-\beta}{\frac{1}{3}a\beta^4}\left(\frac{R}{\mu}\right)^4\left(x^2\frac{d\theta}{dx}\right)^2 \quad . \quad . \quad (18)$$

The value of $x^2 d\theta/dx$ can be read from Emden's table. Taking the entry for the last line, which corresponds to $\theta = 0$, and denoting values at this point by the suffix 1, we have

$$M^2 = \frac{4}{\pi G^3}\cdot\frac{1-\beta}{\frac{1}{3}a\beta^4}\cdot\left(\frac{R}{\mu}\right)^4 x_1^4\left(\frac{d\theta}{dx}\right)_1^2 \quad . \quad . \quad (19)$$

This is Eddington's quartic equation with his numerical coefficient.

The assumptions β = constant and μ = constant determine θ as a function of x and so determine the relative density-distribution. We now calculate how fast this density-distribution is radiating to space and what sources are required to maintain it at a steady temperature.

We simply require the boundary value of $L(r) = 4\pi r^2 F r$. By (4) and (5)

$$\frac{dp'}{dP} = \frac{\kappa L(r)}{4\pi c G M(r)} \quad . \quad . \quad . \quad . \quad (20)$$

or

$$L(r) = \frac{4\pi c G M(r)}{\kappa} \frac{d[(1-\beta)P]}{dP} \quad . \quad . \quad . \quad (21)$$

The luminosity is the boundary value of this as $P \to 0$, which is

$$L = \frac{4\pi c G M(1-\beta)}{\kappa_1} \quad . \quad . \quad . \quad . \quad (22)$$

For a finite luminosity κ_1 must be finite and non-zero. Since $(1-\beta)$ is constant, by (20) $\kappa L(r)/M(r)$ is constant through the star, or in Eddington's notation $\kappa\eta$ = constant. If κ is also constant it is readily found from (6) that ϵ_r is constant. But *we can attribute an arbitrary distribution of* κ *to the interior* (tending to the same value κ_1 at the boundary) and always determine the corresponding source-distribution from (6). The luminosity L remains unaltered.

This is the complete solution for the hypotheses β and μ constant. Such is Eddington's model, and our results are identical with his when his hypotheses are satisfied through the whole star.

(*b*) *β and μ variable in any manner.*—We now remove the restriction that β and μ shall be constant. Then ϕ varies in an arbitrary manner, starting from the value $\phi = 1$ at the centre. Then (14) becomes a definite differential equation for θ, provided ϕ is specified for each value of θ. For example, if $[(1-\beta)/\beta^4\mu^4]^{\frac{1}{4}}$ halved itself from centre to boundary, we could adopt $\phi = 1 - \frac{1}{2}(1-\theta^n)$.

Let (14), with some specified ϕ, be solved with the initial conditions $\theta = 1$, $d\theta/dx = 0$ at $x = 0$. [We can put $x = 2\xi$ if we like to have strict comparison with Emden's equation.] Let the solution be continued till θ vanishes, say at $x = x_1$. This number x_1 will not be the same as for Emden's equation—it will depend on the ϕ chosen. Similarly the values of $d\theta/dx$ will all be different. The specification of ϕ determines θ as a function of x and so determines the relative density-distribution.

We now find the total mass. We have

$$M(r) = \int_0^r 4\pi\rho r^2 dr = 4\pi\rho_c \left(\frac{P_c}{4\pi G\rho_c^2}\right)^{\frac{3}{2}} \int_0^x \theta^3 x^2 dx$$

$$= -\frac{4^{-\frac{1}{2}}}{\pi^{\frac{1}{2}}G^{\frac{3}{2}}} \frac{P_c^{\frac{3}{2}}}{\rho_c^2} \left[\frac{x^2}{\theta^3}\frac{d}{dx}(\phi\theta^4)\right] \quad . \quad . \quad (16')$$

Taking the limit of this equation as $\theta \to 0$, we have

$$M = -\frac{4^{\frac{1}{2}}}{\pi^{\frac{1}{2}}G^{\frac{3}{2}}} \frac{P_c^{\frac{3}{2}}}{\rho_c^2} \phi_1 x_1^2 \left(\frac{d\theta}{dx}\right)_1, \quad . \quad . \quad . \quad (19')$$

where ϕ_1 denotes the value of ϕ at $\theta = 0$. By (7) the value of the constant P_c^3/ρ_c^4 is given by

$$\frac{P_c^3}{\rho_c^4} = \frac{1-\beta_c}{\frac{1}{3}a\beta_c^4}\left(\frac{R}{\mu_c}\right)^4 = \frac{1-\beta_c}{\beta_c^4\mu_c^4}\cdot\frac{\mu_1^4\beta_1^4}{1-\beta_1}\cdot\frac{1-\beta_1}{\frac{1}{3}a\beta_1^4}\left(\frac{R}{\mu_1}\right)^4$$

$$= \frac{1}{\phi_1^3}\frac{1-\beta_1}{\frac{1}{3}a\beta_1^4}\left(\frac{R}{\mu_1}\right)^4 \qquad (17')$$

Introducing into (19′) we get

$$M^2 = \frac{4}{\pi G^3}\frac{1-\beta_1}{\frac{1}{3}a\beta_1^4}\left(\frac{R}{\mu_1}\right)^4\frac{1}{\phi_1}x_1^4\left(\frac{d\theta}{dx}\right)_1^2 \qquad . \qquad . \qquad (19'')$$

This is the generalisation of (19) for arbitrarily variable μ and β. It may be written in the form

$$\frac{1-\beta_1}{\beta_1^4}\left(\frac{R}{\mu_1}\right)^4 = \frac{\pi a G^3 M^2 C}{12} \qquad . \qquad . \qquad . \qquad (19''')$$

where

$$C = \frac{\phi_1}{x_1^4(d\theta/dx)_1^2}$$

C depends solely on the behaviour of $\phi(\theta)$ from $\theta = 1$ to $\theta = 0$, *i.e.* on the relative density-distribution. It will be seen that, given the *form* of the variation of $(1-\beta)/\beta^4\mu^4$ through the star (*i.e.* given ϕ), (19‴) determines the manner in which M fixes the *scale* of $(1-\beta)/\beta^4\mu^4$. For (19‴) determines the boundary value of $(1-\beta)/\beta^4\mu^4$ when ϕ is prescribed, and since ϕ is prescribed the value of $(1-\beta)/\beta^4\mu^4$ at any other point can then be calculated. C can always be found by numerical quadrature of (14) for any prescribed ϕ.

We now calculate how fast the density-distribution prescribed by ϕ is radiating to space. As before we simply require the boundary value of $L(r) = 4\pi r^2 F_r$. Equation (21) holds as before, and letting $P \to 0$ on the right-hand side, so that $\beta \to \beta_1$, we get

$$L = \frac{4\pi c G M(1-\beta_1)}{\kappa_1} \qquad . \qquad . \qquad . \qquad (22')$$

The source distribution can be found from (6), for any arbitrary distribution of interior opacity κ tending to κ_1 at the boundary; for the complete march of $1-\beta$ is now known since $(1-\beta)/\beta^4\mu^4$ has been found everywhere.

Eddington has been accustomed to interpret equations of the type (22′) as expressing a dependence of luminosity on opacity. On our interpretation (22) simply gives the condition that the density-distribution, of mass M, shall radiate to space at a rate equal to the prescribed rate of generation L. Given L, M, and κ_1 (or given T_1 if κ_1 is fixed by some physical law), (22′) determines $1-\beta_1$. Then (19‴) determines the numerical value of C, which is the single restriction imposed on the relative density-distribution ϕ. I have shown in my earlier paper that this is physically equivalent to a fixation of the mean gradient of ϕ between centre $\theta = 1$ and boundary $\theta = 0$.

It will be observed that our equations reduce strictly to Eddington's when β and μ are constant.

I have not proved that a ϕ exists for arbitrary L, M, and κ_1. This is a problem for the pure mathematician, though my pupils are examining it numerically.* But simple physical considerations make it extremely probable that if we start off an aggregate of matter of mass M with arbitrary generation L (not too large) and arbitrary opacity, it will simply cool freely until it assumes a steady state in which the rate of radiation to space equals the rate of generation L. The aggregate (L, M) then possesses a definite ϕ specifying its density-distribution. A more precise form of these considerations was given on p. 21 of my earlier paper.†

(*c*) *Removal of restriction to perfect-gas.*—If the gas is not perfect we cannot employ equation (7). In this case, whatever the equation of state, we define a function ϕ by the relation

$$\phi = \frac{P/P_c}{(\rho/\rho_c)^{\frac{5}{3}}}.$$

It is then found, as shown in my earlier paper, that the analysis can be carried through exactly as for the perfect-gas case, provided only we assume the gas-laws hold in the immediate neighbourhood of the boundary. It is readily found, in fact, that to establish (17′) we only require the gas-laws to hold near the boundary. The two equations (19‴) and (22′), which fix the numerical value of C for given L, M, and κ_1, and the differential equation (14), satisfied by θ for a prescribed ϕ, remain unaltered. The equation of transfer (5) is only required in the outer layers.

4. The only point requiring further discussion is the practical application of the formulæ to an actual star. In my earlier paper I showed that we could interpret the level $x = x_1$, $\theta = 0$ as the level at which $T = T_1$ (the average depth from which the radiation comes), T_1 being the effective temperature; the errors introduced by the difference between the boundary conditions of the model, as thus interpreted, and the true boundary conditions are then very small, as shown by my Table V. As Sir Joseph Larmor ‡ has expounded in detail, the success of the model depends on the smallness of $d \log (P/\rho^{\frac{5}{3}})/d \log \rho$ compared with unity in the effective radiating layers of the star. It is easily verified from the photospheric investigations given in my Bakerian lecture and elsewhere that this condition is satisfied in actual stars, as far as we can ascertain.§

* *Note added* 1930 *May* 25.—Since the above was written I have been able to show that, given any numerical value for C, $a\phi$ can always be constructed such that $\phi_1 x_1^{-4}(d\theta/dx_1)^{-2}$, when computed from (14), takes the value C. This is in accordance with the common-sense view that given an arbitrary M and an arbitrary L (not too large) an equilibrium distribution of mass M can always be found which radiates at the rate L.

† *Monthly Notices*, **90**, 21 (1929). ‡ *Observatory*, 1930 March.

§ The point is that the model in which P and ρ vanish together at the boundary represents the actual star with great accuracy, save outside the photospheric level $T=T_1$. The radiation from these layers outside $T=T_1$ being small, the model

5. If we write our general mass-formula (16′) in the form

$$M(r) = -\frac{4^{\frac{1}{2}}}{\pi^{\frac{1}{2}}G^{\frac{3}{2}}}\frac{P_c^{\frac{3}{4}}}{{\rho_c}^2}x^2(\phi + \tfrac{1}{4}\theta\phi')\frac{d\theta}{dx}$$

and, following Eddington, return to the original variables r, ρ, P, we find

$$-\frac{3}{4}\frac{\rho^2}{P}\frac{GM(r)}{r^2} = \frac{d\rho}{dr}\left[1 + \frac{3}{4}\frac{d\log P/\rho^{\frac{4}{3}}}{d\log\rho}\right]$$

which reduces identically to the equation of mechanical equilibrium. At the boundary $\rho = 0$, and the second term in the square brackets vanishes in the model we are considering. This term was omitted by Professor Eddington in his second paper, but it is quite invalid to integrate unless this term is present.

6. I have now shown that my work involves no mysticism and no sophistries. It involves no unfounded speculation. I have shown complete mathematical linkage between the interior and the quantities I evaluate, in the form of an explicit differential equation * connecting a pure number C, evaluable from observables, with the relative density-distribution of the interior. My paper presents a method of finding out something about the density-distribution in the stellar interior, by observation of the way in which it has adjusted its surface layers so as to be compatible with enclosing the desired mass M and radiating at the prescribed rate of generation L. I make no irresponsible conjectures and no assumptions except those explicitly stated. So far from creating confusion in the subject I claim to have clarified it, by stating and solving the stellar problem in the form in which it presents itself in nature, namely, what are the possible relative density-distributions of a star of prescribed mass M, generating energy at a prescribed rate L? The answer is that as far as steady-state considerations are involved, any distribution ϕ is possible for which the corresponding C takes a particular numerical value depending on the prescribed values of M, L, and κ_1 or of M, L, and T_1. This restriction on ϕ involves no knowledge of the physical properties of the interior, but depends only on the equation of state of the effective radiating layers and the state of radiative equilibrium there.

7. On a future occasion I hope to present numerical values of C calculated from the differential equation (14) for certain definite forms of ϕ, *i.e.* for certain definite density-distributions.

gives very closely the same radiation to space as the actual star. We identify the boundary values for the model with the values for the level $T=T_1$ in the actual star. Professor Eddington's allegation that this process is an "oversight," which "vitiates my paper (p. 285) so far as its bearing on the stellar interior is concerned," is fully met by my specific investigations of the point.

* It is unaccountable that Professor Eddington should have overlooked that C is defined by means of the differential equation (20) of my former paper, which fixes the internal relative density-distribution. C is thus the same for all members of a homologous family.

The Galactic Rotation Effect in Late Type Stars.
By R. O. Redman, M.A.

1. *Introduction.*—From its shape and from general analogy with other celestial bodies it has long been suspected that the Milky Way system is in rotation. This more or less vague idea has in recent years been put on a firmer basis, and the conception of galactic rotation, associated chiefly with the names of Lindblad and Oort, is rapidly becoming one of the accepted theories of general astronomy. Oort's work,* where the theory is linked with observational evidence, and the strong support recently provided by Plaskett and Pearce,† have indeed helped, to a very great extent, our understanding of the motions of certain stars. With the radial velocities of objects of small peculiar motions, O and B stars, the interstellar calcium, *c*-stars and the Cepheid variables, and with stars of very great velocities, the galactic rotation theory has been very successful. These, however, are all peculiar or rare objects in space. When we turn to cooler stars difficulties appear.

Some of these difficulties are merely observational. As is now well known, the "galactic rotation" effect which is sought in radial velocities and in proper motions is a variation of the nature of a second harmonic in galactic longitude, and it also depends directly on the average distance from us of the objects whose movements are considered. Clearly it is easier to detect and estimate the size of this effect in the case of distant objects of high luminosity and small peculiar motion, than with nearer, fainter stars such as the late type giants whose peculiar velocities are large. Most of the suitable observational data at present available for the latter are found in proper motions. With these we have two additional difficulties. Systematic errors are likely to vary with right ascension and give a spurious galactic rotation effect on analysis, and any error in the precession constant will also affect all results. The total elimination of these two difficulties is by no means an easy task. Finally, radial velocities have not been determined for any appreciable number of these stars fainter than apparent magnitude 5·5—that is, for F—K giants at distances greater than 100 to 150 parsecs.

Besides these observational difficulties, there has been another point at which late type stars have not accorded so well with theory as those of greater luminosity and slower motions. From the term $r\mathrm{A}\sin 2(l-l_0)$ in radial velocities, or $[r\mathrm{A}\cos 2(l-l_0)+r\mathrm{B}]$ in proper motions, it is possible to deduce a value for l_0, the longitude of the centre of the galaxy. The B stars, interstellar calcium, etc., point to a centre at longitude 325° or 330°, agreeing with independent evidence, *e.g.* that from the globular clusters. Late type stars, on the other hand, give l_0 from 10° to 30° away. For instance, Schilt ‡ has derived three values from three different sources of proper motions and gets $l_0 = 351°$, 340°, and 337°. J. E. Merrill,§ using motions derived from the Yale

* *B.A.N.*, 120, 132, 133 (1927), 159 (1928).
† *M.N.*, **90**, 243, 1930.
‡ *A.J.*, **39**, 20, 1928.
§ *A.J.*, **39**, 90, 1929.

50°–55° catalogue, gets $l_0 = 348°$ and 0° from two solutions differing in detail. Oort from radial velocities for types A–G, gets $l_0 = 337°$ and 345° in two determinations.

Thus it appears very necessary that more information should be obtained concerning the relation of late type stars to the "galactic rotation" effect. The determination of radial velocities of such stars offers one line of attack. The method is slow but more reliable than those which use proper motions. It is desirable to consider objects as distant as is convenient, and it is best, if possible, to avoid stars of variable radial velocity. The K giants seem to form the most suitable class to consider; they are numerous and on the whole probably as little subject to variation as any stars in the sky, while those variables which do exist are likely to have a small range in radial velocity.

2. *Observations.* — Thus it was decided to observe K stars of apparent visual magnitude 7·0 to 7·5, within 10° of the galactic equator and north of declination − 10°. This last restriction was necessary since the work was to be done at the Dominion Astrophysical Observatory, Victoria, B.C. However, since these cover only right ascensions, 17^h to 8^h, more stars of the same magnitude were added from 8^h to 10^h and 15^h to 17^h, up to galactic latitude 45°. A few near the galactic pole were also observed. All the required data for position, magnitude, and type were taken from the H.D. catalogue. The list as originally drawn up contained an unknown number of dwarfs. Reference to proper motions showed these to be comparatively few; about half a dozen stars of large motion were struck off and also one or two binaries.

The object of these observations being essentially statistical, individual velocities with a high degree of accuracy were not required, and only one plate was taken of each star. The standard Cassegrain spectrograph on the 72-inch reflector at Victoria was used with one prism and a short focus camera. This arrangement gave spectra with a dispersion of about 90A per mm. at $H\gamma$, which is very small for a radial velocity programme. Observations were commenced in 1928 November and, as far as this paper goes, continued until 1929 December. 209 measurable plates were obtained in or near the galaxy and 36 near the galactic pole.

3. *Measurement.* — Tests showed that measurements of these spectra with the standard micrometer were more accurate, though slower, than those with the spectro-comparator. The micrometer was therefore used throughout. Spectra were taken of standard (giant) stars, and from these wave-lengths were obtained which were used throughout this work. About 25 lines were used in obtaining each velocity. At various times throughout the year spectra (26 in all) of other standard stars were photographed and measured to check the wave-lengths and the general method of procedure. The mean residual (with sign) from these 26 spectra was − 0·5 km./sec., while the probable error per plate was found to be ± 3·0 km./sec.

Dwarf stars did not give such good results. In their case the wave-lengths obtained, as mentioned above, appear to have a systematic error

which makes the observed velocities 4 or 5 km./sec. more negative than they should be. The cause of this is obscure, the only reason which can be suggested is that the blends formed by the spectral lines alter a little with absolute magnitude. It must be remembered that with this small dispersion practically all observed lines are blends of two or more single lines of considerable strength. Since we are in general observing only giants, this systematic error is not of great importance.

A difficulty arises when one wishes to determine the probable errors of the velocities obtained. For the galactic stars only one plate of each was obtained, so that inter-comparisons were impossible. Three stars were selected at the galactic pole, H.D. 107469, 107854, and 116515, and respectively 8, 7, and 7 spectrograms were obtained. It was found later that the first of these is unfortunately a dwarf. However, from the inter-agreement of the plates, "probable errors" 2·9, 5·1, and 2·7 km./sec. were found. The mean of these is 3·6. Further, a camera used extensively on the same spectrograph gives spectra of about 3·5 times the dispersion used in this work, and for late type stars is considered to give velocities with a probable error of about 1 km./sec. In comparison, therefore, we would expect our spectra to give velocities with probable errors about 3·5 km./sec.

Since practically none of the stars investigated has had its velocity determined at other observatories, there is at present no external check on the results obtained here. The evidence given above, from standard stars, from the three stars at the galactic pole, and from comparison with spectra of another dispersion, goes to show that the average "probable error" is between 3 and 4 km./sec. For the present, then, 3·5 km./sec. is taken to be the average "probable error" per plate. The "probable error" of the "probable error" may be estimated as $\pm$ 0·4 km./sec. This last figure has not been obtained by any precise calculation; rather it is of the nature of a personal opinion.

4. *Elimination of Dwarfs.*—Since a few dwarfs with small transverse velocities may have escaped elimination by proper motion, and since their presence in this work is undesirable, an attempt was made to detect them by their spectra. Accordingly, a dozen standard dwarf spectra were taken and compared with those of standard giants. Inspection showed that many of the criteria for spectroscopic absolute magnitudes were of little or no value with the small dispersion employed. The intensity ratios of λ4215 to λ4250 and λ4455 to λ4462, and, to some extent, the relative intensities of $\lambda\lambda$4482, 4489, and 4495 were found useful. Only a small minority of the programme stars showed "dwarfish" features, and only two or three appeared to be dwarfs beyond doubt.

5. *Results and Discussion.*—Velocities were available for 225 stars, 210 in or near the galaxy and 15 near the galactic pole. In the calculations for solar motion, galactic rotation, etc., 24 stars were rejected, the criteria for rejection being: total annual proper motion $> 0''{\cdot}1$, or observed velocity $>$ 100 km./sec., or "dwarfish" spectrum. It may be noted that four stars were found to have velocities greater than 100 km./sec. and, of these, two show moderate proper motion

($> 0''{\cdot}1$ per year), and all four have giant spectra. Finally, there were left 188 galactic stars and 13 at the galactic pole. These were collected into 13 groups, the mean galactic co-ordinates and the mean velocity being found for each group, and with these values solutions were made for the solar motion and "galactic rotation." The method was the same as that used by Plaskett.* Eight different solutions were made and the results are tabulated below. V_0 is the solar velocity (in km./sec.) and L and B are the galactic co-ordinates (in degrees) of the solar apex. $\bar{r}A$ is the usual term associated with galactic rotation while l_0 is the longitude of the galactic centre. Assumed figures are in brackets, and the "probable error" is given with each calculated quantity.

Solution.	K.	V_0.	L.	B.	$\bar{r}A$.	l_0.	Average Group Residuals. Without Sign.	Average Group Residuals. With Sign.
1	(0)	18·6±2·2	(22)	(20)	3·8±1·7	356±17	5·3	−5·3
2	−5·9±0·7	15·6±1·1	(22)	(20)	3·8±1·1	350±7	2·7	0·0
3	(0)	(20)	(22)	(20)	4·4±2·0	358±13	5·3	−5·1
4	−5·2±0·2	(20)	(22)	(20)	5·4±1·3	358±7	3·1	0·0
5	(0)	17·6±2·0	(22)	(20)	2·7±1·8	(330)	5·2	−5·2
6	(0)	21·7±1·5	50±4	20±14	5·4±1·4	18±7	3·5	−0·1
7	(0)	19·4±1·7	47±4·5	26±9	(0)	(0)	3·7	−1·5
8	(0)	(19·4)	(47)	(26)	3·4±1·2	22±10	3·3	−2·1

In the first five solutions a solar apex was assumed (at $\alpha = 271°$, $\delta = +28°$) in order to reduce the number of unknowns to a minimum. It was found that, unless a K term was included, values for V_0, $\bar{r}A$, and l_0 could not be found so as to furnish residuals not systematically negative. Since such a term has not hitherto appeared in the velocities of late type stars and, indeed, is no longer regarded as an indispensable feature of the motions of B stars, it seemed undesirable to consider it in this work. The alternative is either to admit a systematic error in the velocities or to reject the apex at (271°, + 28°). The latter seems to be the correct course to pursue in view of the absence of systematic error in the tests with standard giant stars (§3). Thus in solution (6) we find an apex at L = 50°, B = 20°, corresponding to $\alpha = 283°$, $\delta = +53°$. This is at some distance from our other assumed apex, but, despite the large "probable error" of B (due to the distribution of the stars in the sky), we find in proper motion work plenty of supporting evidence.

For instance, Dyson and Thackeray,† using the Greenwich catalogue, covering the zone 24°–32° N. declination, find apices as follows:—

K0 (1221 stars)	$\alpha = 273°$	$\delta = +51°$
K2 (392 ,,)	293°	+59°
K5 (229 ,,)	318°	+65°.

* *M.N.*, **88**, 395, 1928.

† *M.N.*, **79**, 201, 1919.

Strömberg* finds for K giants ($M \leqslant 4$) a solar apex at $\alpha = 282°$, $\delta = +43°$, with V = 18·3 km./sec. In a recent analysis of proper motions,† Schilt and Miss Barney have remarked that there is strong evidence that the solar apex for giant G and K stars is at a higher declination than for dwarf stars belonging to the " main series." There is thus no reason to doubt the reality of our result. It is to be noted that in solution (7), where galactic rotation is not considered at all, the solar apex is at L = 47°, B = 26°, *i.e.* at $\alpha = 273°$, $\delta = +52°$, in almost exact agreement with the value given by Dyson and Thackeray for Ko stars. An important fact is that in this solution the residuals are not very much greater than in solution (6) and, further, do not show any great systematic trend. The addition of rotation terms to the solution (in (8)) does not make any appreciable improvement, but this is merely an indication that if both solar motion and galactic rotation are to be considered, they should be treated simultaneously as in (6).

Taking our true group velocities to be defined by the elements in solution (6), we find the average residual per star is 19·5 km./sec. This is practically the same as the average peculiar radial velocity computed from the mean velocity of each group. These K stars evidently have large peculiar motions and, owing to this, each of the observed group velocities is subject to an average " error " of 5 km./sec. We note that as far as mere numerical size is concerned, only two of the solutions above yield average residuals greater than this figure. Without more data our results are clearly subject to considerable uncertainty.

The quantity $\bar{r}$A, whenever included, has a value between 2·7 and 5·4 km./sec. Let us take the actual value to be 4 km./sec. Estimates of A have been made by Oort and by Plaskett. As far as present knowledge goes, the best value is 0·017 km./sec. per parsec, which would show $\bar{r}$ to be about 240 parsecs.

For comparison the mean parallax of these stars was found, using all available proper motions. The stars so used were not quite identical with those whose radial velocities had been determined, but were all on the original observing programme, and thus may be taken as homogeneous with those observed. Rejecting proper motions greater than 0″·1 per year, we have left 191 stars, and taking data from solution (6), viz. solar apex at $\alpha = 283°$, $\delta = +53°$, solar velocity 21·7 km./sec., and average peculiar velocity 19·5 km./sec, a mean parallax 0″·0047 was found. The distance corresponding to this is about 210 parsecs; it should be rather smaller than the true mean distance, so that the 240 parsecs obtained from galactic rotation may be considered to agree very satisfactorily.

The mean magnitude corresponding to the mean parallax is $+ 0^{m}·6$, and corresponding to the mean distance from galactic rotation it is $+ 0^{m}·35$. The true mean magnitude should be between these two values. This agrees with other evidence, for instance, that from the hypothetical parallaxes of binary stars.‡

* *Mt. Wilson Contributions*, **11**, 300, 1922.
† *A.J.*, **37**, 189, 1927.
‡ *M.N.*, **88**, 722, 1928; or Russell and Miss Moore, *A.J.*, **39**, 165, 1929.

Now although the $\bar{r}$A term is in comfortable agreement with other work, the case is otherwise with the value of l_0 which we obtain. An assumed value 330° (solution (5)) is clearly unsuitable. The other values range from 350° to 22°, our most satisfactory solution (6) giving 18°. We have seen earlier that a similar difficulty arose with the radial velocities of comparatively near late type stars, and that proper motion work also gave an l_0 (337° to 0°) differing considerably from the usually accepted value near 325° or 330°. Since all investigations with late type stars have given a similar result, it is most probable that the effect is real.

It seems probable that the phenomenon is intimately connected with star-streaming. The B stars, interstellar calcium, etc., which furnish us with the expected l_0 are not subject to star-streaming. On the other hand, most late type stars, and in particular the K stars investigated here, show directions of preferential motion. From a theoretical point of view * it is to be expected that the direction of stream motion would coincide with the direction of the galactic centre. The observed apices of star-streaming (removing solar motion) are usually taken to be near galactic longitudes 167° and 347° and practically on the galactic equator. This is not the direction of the usually accepted centre, but on the contrary it agrees much better with the l_0 from the " galactic rotation " effect in these cooler stars.

Schilt, who has been responsible for much of the proper motion investigations concerning galactic rotation, has recently published a paper † in which he considers the second harmonic term to be real (he takes it in right ascension rather than in galactic longitude), but rejects the rotation theory and suggests that the observations may be explained entirely by stream motion. The matter is not fully explained, but we are promised another paper on the subject. The still more recent work of Plaskett and Pearce, already mentioned, together with the evidence collected by Oort, makes the entire rejection of the rotation theory very difficult.

The complicated movements of the cooler stars form a more difficult problem than that furnished by the slow-moving B stars. A complete theoretical explanation is not yet available, but more observational data would no doubt help greatly our understanding of the matter, and the writer hopes to be able to continue the work outlined here. The method of using a small dispersion spectrograph and taking only one photograph of each star seems to be the best one to follow, since individual velocities are here of little interest. The chief danger lies in systematic rather than accidental errors. Until a fair number of independent observations from elsewhere are available the use of bright standard stars as checks seems to be the only possible safeguard.

6. *Summary.*—A description is given of the observation and analysis of the radial velocities of some two hundred seventh magnitude K stars situated near the galactic equator. A small dispersion spectro-

* Oort, *B.A.N.*, 159, 1928.
† *A.J.*, **39**, 143, 1929.

graph was used and only one plate was taken of each star. The average "probable error" per plate is 3·5 km./sec. Solutions for the solar motion and "galactic rotation" confirm the high declination of the solar apex and high value for l_0, the longitude of the galactic centre, found with late type stars by other workers, who have used principally proper motions. The "$\bar{r}A$" term agrees satisfactorily with other estimates. The average peculiar radial velocity of these stars is about 20 km./sec. (four were found > 100 km./sec.) and there are definite signs of stream motion. It is significant that in all work on late type stars, the "l_0" agrees better with the apices of stream motion than with the usually accepted galactic centre.

I wish to thank the members of the observatory staff for the interest they have shown in this work; in particular the Director, J. S. Plaskett, who offered the original suggestion leading to the investigation, and has helped with advice on many points; also J. A. Pearce and C. S. Beals, who obtained some of the plates. I am also indebted to Professor Schlesinger who kindly furnished the proper motion data of the stars used.

A more complete account of this work will appear in the Publications of the Dominion Astrophysical Observatory.

Victoria, B.C.:
1930 *March.*

Periodic Variation in the Colours of the Two Equatorial Belts of Jupiter. (Second Paper.) By A. Stanley Williams.

The first paper on this subject was published in the *Monthly Notices*, **59**, 376, 1899. The observational data on which the conclusions depended consisted entirely of verbal descriptions of the colours of the two belts by a great many different observers. Some time previously Julius Schmidt of the Athens Observatory, the well-known observer and author of the great map of the moon, had used a numerical scale, ranging from 0 to 10, to express the degree of redness of certain stars; a similar scale of redness was adopted for indicating the redness of the belts, and the verbal descriptions of the colours by the various observers were converted into terms of this scale of redness. The colour of the great red spot, when at its maximum degree of redness, was assumed to be equal to 10 of this scale, 0 implying an entire absence of red colour.*

The observations of the various observers are usually remarkably accordant for the years when the belts were near a maximum or minimum of redness, notwithstanding the difficulty of converting verbal descriptions into terms of a numerical scale. This will be seen from the following results for 1879, when the S. equatorial belt (SEB.) was at

* No account was taken of bluish or greenish tints, although, strictly speaking, they should be considered to represent a negative or minus degree of redness.

or near its minimum redness, and the N. equatorial belt (NEB.) at or near its maximum.

S. Equat. Belt.		N. Equat. Belt.		Observer.
Redness.	Obs.	Redness.	Obs.	
0·7	12	3·5	27	Niesten and Stuyvart.
0·2	39	4·5	39	Bredichin.
0·7	3	4·7	4	O. Lohse.
0·0	8	5·9	8	Terby.
..	..	6·0	1	Brett.
0·0	1	6·0	1	Knott.
0·0	2	5·5	2	Dennett.
0·0	1	7·0	1	"I. V. T."
0·0	..	6·0	..	Barnard.
0·0	9	5·4	10	Williams.

The mean values are 0·2 for the SEB. and 5·4 for the NEB. The figures are, of course, only my figures converting the verbal descriptions of the colours as given by the various observers into the adopted scale of redness. But it is quite impossible to draw any other conclusion from the observations than that whilst the NEB. was deep red in 1879, the SEB. was practically devoid of all red coloration. To give the individual observations would occupy far too much space and would subserve no adequate useful purpose, but the following description given by so eminent an observer as the late E. E. Barnard may be quoted.*

Speaking generally, Barnard remarks, "When I first examined the belts in 1879, the northern one was reddish, whilst the southern was bluish." And more particularly he states that "North of the narrow light rift in the northern part of the north belt, the color was frequently of a deep, rich vermilion, while the rest of the belt towards the equator was of a much lighter red, though at times it became a very deep darkish red. The south belt remained bluish for a long time, and I first began to call it reddish about the middle of August 1880. Even in September I have called it a drab color." Barnard was at the time paying much attention to Jupiter, and his conclusions are evidently the summation of a great many observations. In addition to his observations there are no less than 168 by the other nine observers.

In the year 1893 my own observations of the colours of the belts assumed a more natural and systematic character. The degree of redness according to the scale of 0–10 was now actually estimated at the telescope. The redness of the belts has now been recorded in this manner on 444 nights up to the present time.† They thus cover a period of nearly forty years, and since the observations were all made

* *Pub. Ast. Soc. Pacific*, **1**, 98, 1889.

† On many nights two or three observations were made in different longitudes of the planet.

by one and the same observer, they should be strictly comparable *inter se*. Moreover, with a few unimportant exceptions, the same telescope, a 6½-inch reflector, was used throughout, and even usually the same eyepiece. Unfortunately, the 444 observations are not uniformly spread over the period of observation, but in some years they are only a few in number, and in two or three years there is not one. This defect is largely owing to the difficulty of making satisfactory observations of colour in the years when Jupiter is at its extreme declination south. These years may be designated as lean years. But the years immediately preceding and following a lean year are not much better. The lean years are given at the foot of the diagram (fig. 1). Now that so much more attention is being paid to astronomical matters in the southern hemisphere, it is to be hoped that more observations will be forthcoming from the observers in that hemisphere in the years when the planet is in south declination.

Table I. gives the yearly means of the observed redness of the two equatorial belts from my observations since the year 1893, when, as already stated, the redness was directly recorded at the telescope in terms of the scale of redness, 1–10. The table is sufficiently self-explanatory. The chief uncertainty is caused by the difficulty in fixing an upper limit to the scale of redness, since markings having a redness equal to 10 are only rarely visible. Coloured discs are of little use, because the actual kinds of redness are so various. The observed redness is the integrated redness of the belt as a whole. The presence of many bright rifts or spots in a belt, as happened with the NEB. in 1927, would reduce its degree of redness as a whole. It might perhaps have been better not to have taken account of the presence of bright rifts or spots, but to have limited the observations to the dark parts of the belts only. Sometimes bluish or greenish tints have been noted, as already mentioned. As already stated, it would seem that, strictly speaking, such observations should have been treated as constituting a negative degree of redness, but actually no account has been taken of bluish or greenish tints.

Notes.—No. 8, There was a decided decrease in the redness of SEB. during the apparition, and also an increase in that of the NEB., which was narrow and simple. 10, NEB. was a fine, bright, deep red colour, whilst the SEB. usually was not red. The belts were finely contrasted. 12, The NEB. was generally too faint and narrow for colour, which was only observed on two nights. The mean date for the NEB. is 1905·86. 15, Both belts described as brownish, rather light, with an ochrey tinge. The NEB. was probably rated too high. 23, There was a decided decrease in redness of the NEB. from 1919 October onwards. 29, The mean redness, 4·1, of the NEB. is certainly underrated. There were many white rifts and spots in the belt. The SEB. was bluish grey, and the bluish tinge was not due to contrast with the red NEB., since the former still appeared bluish when the latter was concealed by a bar. 31, In the course of the observations of this year the SEB. increased considerably in redness, whilst there was also a decided decrease in the redness of the NEB.

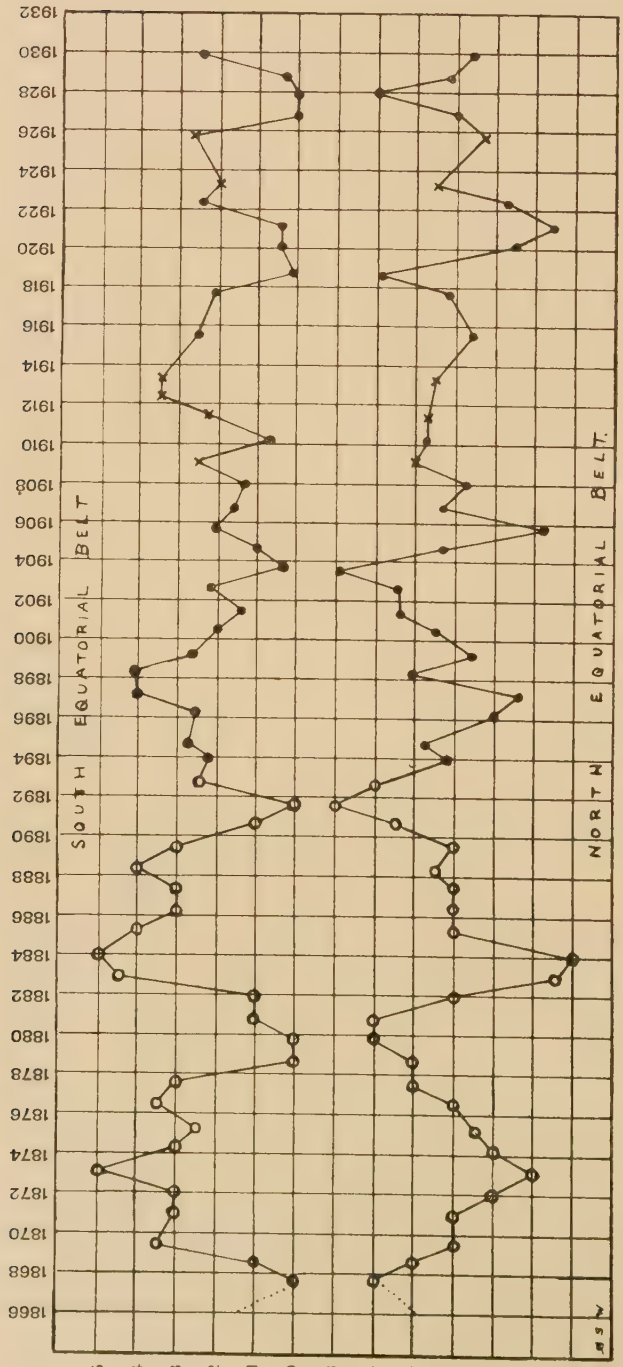
TABLE I.

Observations.

No.	Date.	Redness of SEB.	Redness of NEB.	Nights.	Remarks.
1	1893·97	2·2	3·2	5	
2	1894·72	2·8	3·8	4	
3	1896·23	2·6	2·0	5	
4	1897·21	4·0	1·4	9	SEB. rust-red.
5	1898·24	4·1	4·1	11	Both belts garnet.
6	1899·34	2·7	2·7	25	Both belts red-brown.
7	1900·46	2·0	3·5	13	Both belts rust-red.
8	1901·52	1·4	4·4	25	See note.
9	1902·63	2·2	4·5	27	
10	1903·71	0·3	5·9	33	See note
11	1904·69	1·0	3·3	31	
12	1905·83	2·1	0·8	22	See note.
13	1906·80	1·7	3·3	31	
14	1907·97	1·3	2·8	6	
15	1909·15	2·5	4·0	2	See note.
16	1910·23	0·7	3·8	4	
17	1911·39	2·3	3·8	2	
18	1912·38	3·5	..	1	NEB. narrow and faint.
19	1913·32	3·5	3·5	1	$2\frac{3}{4}$-inch refractor.
20	1915·57	2·6	2·6	7	$2\frac{3}{4}$-inch refractor.
21	1917·73	2·1	3·2	7	
22	1918·74	0·1	4·9	31	NEB. a fine deep red.
23	1920·15	0·4	1·5	26	See note.
24	1921·07	0·4	0·6	7	Very little colour.
25	1922·27	2·4	1·8	6	
26	1923·29	2·0	3·5	1	NEB. chocolate brown.
27	1925·69	2·7	2·3	3	
28	1926·84	0·0	3·0	9	SEB. bluish.
29	1927·88	0·0	4·1	18	See note.
30	1928·81	0·3	3·2	24	
31	1929·90	2·5	2·7	49	See note.

The observations of Table I. are shown in diagram form in fig. 1,* in which all the good determinations (from the observations of more than three nights) are indicated by dots, and the poor determinations

* The observations from 1899 to 1915 may be compared with the figures derived independently by the Rev. T. E. R. Phillips and published in the *Journal* of the B.A.A. for 1915 October.



● = Good Observations (more than three nights).
× = Poor Observations (three nights or less).
○ = Old Observations.
Lean Years, 1865, 1877, 1889, 1901, 1913, 1925.

FIG. 1.—Colour Changes of Jupiter's Belts.

(three nights or less) by crosses. The importance of making observations on a good many nights during an opposition cannot be too strongly insisted on, because not only are there sometimes large differences in different longitudes of the planet, but considerable changes may even occur in the course of the same opposition. In order to make the diagram more complete, the observations between 1867 and 1893, when the present series commenced, have been added from the first paper, and have been indicated by circles. The very remarkable early series of observations by Gruithuisen has been omitted, as its inclusion would have unduly extended the length of the diagram.

Inspection of fig. 1 will show better than words the regular manner in which the maxima and minima of each belt correspond, nearly, with the minima and maxima of the other, resulting in the formation of a really remarkable series of lozenge-shaped interspaces. There is some irregularity in the latter part of the diagram, but not enough to throw out the regular succession of lozenge-shaped interspaces. This irregularity is referred to more particularly later on. Advantage has been taken of the succession of lozenge-shaped interspaces for deriving the period of the colour variations.* The centres of the lozenges were carefully measured from the diagram, and, with the addition of one from Gruithuisen's early observations, there resulted six of what may be termed standard epochs, particulars of which are given in Table II.

TABLE II.

Standard Epochs.

E.	Standard Epoch.	O−C. (I.)	O−C. (II.)
		y.	y.
0	1836·0	+0·25	+0·2
3	1873·3	−0·02	0·0
4	1885·5	−0·34	−0·3
5	1897·7	−0·66	−0·6
6	1912·2	+1·31	+1·4
7	1923·1	−0·31	−0·2

The following exact elements of variation were derived from the above standard epochs.

$$\text{(I.) Standard Epoch} = 1835{\cdot}75 + 12^{y}{\cdot}523\ \text{E.}$$

But for all practical purposes the less precise elements (II.) are sufficiently accurate, and have been adopted, namely,

$$\text{(II.) Standard Epoch} = 1835{\cdot}8 + 12^{y}{\cdot}5\ \text{E.}$$

The residuals O-C according to the two sets of elements are given in Table II. They seem to be quite satisfactorily small. The standard

* The times when the belts are equally red are not so suitable for this purpose as are the times of maximum and minimum redness, or of the standard epochs.

epochs correspond nearly to a maximum redness of the SEB. and a minimum redness of the NEB. But there are differences.

The dates of maximum and minimum redness, derived from the diagram (fig. 1), are given below in Table III. Generally, though not always, they correspond to the highest and lowest peaks and depressions of the curves. In the table, the first column gives the E. reckoned from the first observed maximum or minimum ; the second the observed date of the maximum or minimum ; the third the latter date reduced to E.0 with the period of 12·5 years ; and the last column the differences O–C.

Table III.

Observed Maxima and Minima.

Max. Red. of SEB.

E.	Obs.	Red.	O−C.
0	1836·4	1836·4	+0·15
3	1873·2	5·7	−0·55
4	1884·0	4·0	−2·25
5	1898·4	5·9	−0·35
6	1913·0	8·0	+1·75
7	1925·0	7·5	+1·25

Mean date 1836·25.

Min. Red. of SEB.

E.	Obs.	Red.	O−C.
0	1843·3	1843·3	+1·23
2	1867·5	2·5	+0·43
3	1879·3	1·8	−0·27
4	1891·7	1·7	−0·37
5	1904·0	1·5	−0·57
6	1918·5	3·5	+1·43
7	1927·7	0·2	−1·87

Mean date 1842·07.

Max. Red. of NEB.

0	1843·5	1843·5	+1·31
2	1867·5	2·5	+0·31
3	1880·3	2·8	+0·61
4	1891·5	1·5	−0·69
5	1903·5	1·0	−1·19
6	1918·5	3·5	+1·31
7	1928·0	0·5	−1·69

Mean date 1842·19.

Min. Red. of NEB.

0	1836·3	1836·3	+1·35
3	1872·2	4·7	−0·25
4	1884·0	4·0	−0·95
5	1897·2	4·7	−0·25
6	1911·4	6·4	+1·45
7	1921·1	3·6	−1·35

Mean date 1834·95.

The residuals are interesting and, as might be expected, are larger than those of the standard epochs, partly, perhaps, because the extreme peaks and depressions do not represent in the best possible manner the real maxima and minima. A similar problem frequently confronts us with respect to the maximum and minimum brightness of many variable stars. The standard epochs are really more reliable.

The period of 12·5 years is unexpectedly large, quite considerably longer than the period of Jupiter's revolution round the Sun (11·86 years), but by no justifiable alteration can it be made appreciably less, so as to correspond with the latter. The omission of the early observations of Gruithuisen would not improve matters. Recently Mr. W. A.

Luby has made an interesting and praiseworthy attempt to explain on a physical basis some of the phenomena of the belts and other features of Jupiter, including the present colour periodicities.* His explanation would appear to require equality of the colour periodicity with that of the planet's revolution round the Sun, or else possibly some modification of the initial assumptions on which it is founded.

The unexpectedly long period of 12·5 years is partly due to the fact that the maxima and minima about the year 1918 occurred abnormally late. This was only partially compensated by the last observed maxima and minima about 1928 being early.† It seems likely that the next epoch of maximum and minimum may also be early, since there have been rapid changes in the redness of the two belts during the past opposition. This might further compensate matters.

Whilst these differences may be only accidental, or due in part to errors of observation, yet the residuals are suggestive of the presence of a secondary periodicity affecting the main periodicity, assumed to be 11·86 years. This question must be left to the remote future for elucidation.

Judging from fig. 1 it would seem that the observed range of variation since my systematic observations began in 1893 is less than it was formerly, but it is more likely that I have been underestimating the degree of redness, as compared with the observations of other observers in general. This was certainly the case with respect to the NEB. in 1928, and I have not hesitated to increase the redness of 4·1 as given in Table I. to 5 in fig. 1.‡ The belt contained many white rifts and spots in this year, and this would have had the effect of reducing the degree of redness of the belt as a whole. It would have been better, perhaps, as I have said, to have observed the dark parts of the belts only for redness.

The redness of the NEB. about 1912 was not well observed, the belt being very narrow or faint, whilst 1913 was a lean year. Nevertheless the results of the four consecutive years agree in indicating a moderate degree of redness during this period. The observations by the members of the British Astronomical Association, summarised by the Director of the Jupiter Section (Phillips) in his annual reports, are confirmatory, but indicate a rather less degree of redness.

Table IV. below, and fig. 2, give the mean values of the redness of the two belts derived by means of the elements (II.), or, in other words, the mean colour curves of the belts. The figures in the table, and the dots in fig. 2, are the means of six or seven of the yearly observations in Table I., the data contained in the first paper being included. The most interesting feature, perhaps, is the dissymmetry of the two curves. The flatness of the maximum of the SEB. is noteworthy. But the

* *Popular Astronomy*, **37**, 505, 1929.

† The irregularities of the succession of lozenge-shaped interspaces in the latter part of fig. 1 are due to the same causes.

‡ In all other cases the figures of Table I. have been inserted unaltered in fig. 1.

most remarkable feature is the rapid and continuous decline in redness of the NEB. after maximum. This peculiarity is also brought out strikingly by the individual curves of fig. 1. The actual minimum is followed at first by an equally rapid rise, succeeded by a standstill (or secondary minimum) before the final rise to maximum.

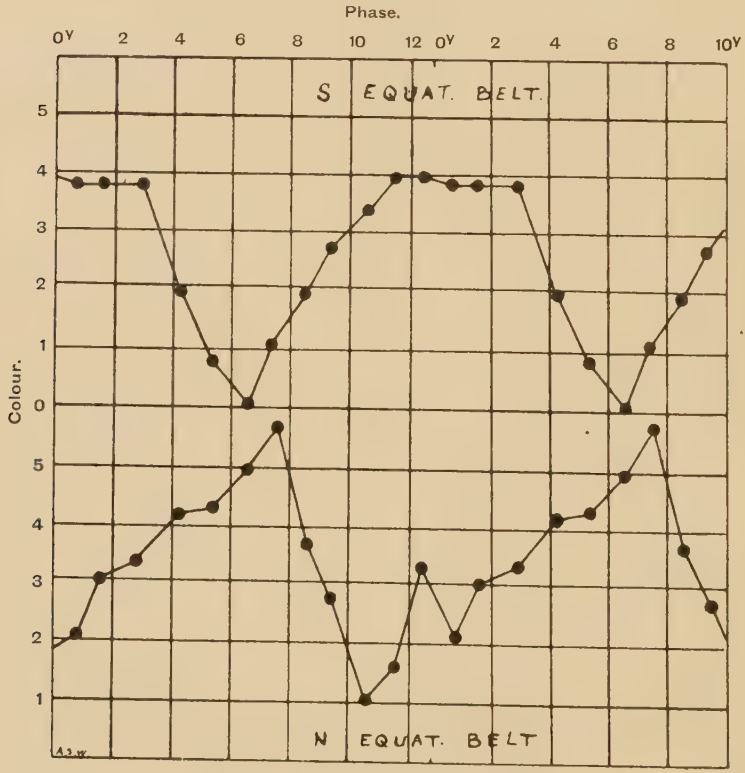


Fig. 2.—Mean Colour Curves of Belts.

Table IV.

Mean Redness of the Belts.

Phase. y.	SEB.	NEB.	Phase. y.	SEB.	NEB.
0·72	3·8	2·1	7·39	1·1	5·7
1·72	3·8	3·0	8·38	1·9	3·75
2·90	3·8	3·3	9·34	2·75	2·7
4·16	1·9	4·2	10·45	3·35	1·05
5·27	0·8	4·35	11·35	3·95	1·65
6·44	0·05	4·95	12·28	3·95	3·3

This rapid decline from maximum redness of the NEB. is quite a marked feature, and there can be no doubt of its reality. During the past opposition there has been such a decided decrease in the redness of this belt that there is every prospect of a repetition of the phenomenon occurring, perhaps during the next year.

Summary.

Standard Epoch = $1835{\cdot}8 + 12^{y}{\cdot}5$ E.
Maximum Redness of SEB. = $1836{\cdot}25 + 12{\cdot}5$ E.

The minimum redness of NEB. precedes this by 1·3 years.*

Maximum Redness of NEB. = $1842{\cdot}19 + 12{\cdot}5$ E.

The minimum redness of SEB. precedes this by 0·12 year.*

The next computed standard epoch is 1935·8, when the SEB. should be near a maximum of redness and the NEB. near a minimum, though the actual minimum redness of the latter will probably be earlier. It is to be hoped that observers will endeavour to observe this important epoch well, and that many observations will be forthcoming from the southern hemisphere, seeing that 1937 will be one of the lean years.

Polvarth, St. Mawes:
1930 *April* 23.

Observation photographique de l'objet Lowell, 1927.
Observatoire royal de Belgique à Uccle.

(*Communicated by Dr. Crommelin.*)

Astrographe triplet Zeiss.
Observateur : E. Delporte.
Recherche de l'objet Lowell.
Plaque : Imperial " Eclipse."
Ciel transparent.
Date : 1927 *janvier* 27.
Pose : durée totale : 64 minutes.
Milieu de la pose : $9^{h}\ 27^{m}\ 41^{s}$ t.m.Gr.
Centre de plaque :

ζ Geminorum, $\alpha = 6^{h}\ 59^{m}\ 47^{s}$, $\delta = +20^{\circ}\ 40'\ 44''$.

Objet ?? Eclat 15^{m} à $15^{m}{\cdot}5$.
Raccord à l'étoile B.D. + 21°, n° 1503, gr. $8^{m}{\cdot}4$ = Abbadia, n° 4021.

Objet − étoile = $\begin{cases} +\ 38^{s}{\cdot}95. \\ +\ 11'\ 33''{\cdot}7. \end{cases}$

Objet 1927·0 $\begin{cases} 7^{h}\ 1^{m}\ 59^{s}{\cdot}7 \\ +\ 21^{\circ}\ 17'\ 59''{\cdot}5. \end{cases}$

* Making use of the data of Table III. The mean colour curve would give a larger interval.

L'objet a été trouvé par comparaison au blink-microscope Zeiss de la plaque ci-dessus avec une autre plaque prise le 1929 janvier 14 avec une plaque impériale 1200 H.D. d'une durée de pose de 58 m., avec comme centre de plaque $\alpha = 6^h\ 53^m\ 55^s$, $\delta = +22°\ 34'$.

Les intensités des deux plaques sont de même ordre.

Une autre plaque du 1929 janvier 28 ayant 61 m. de pose, prise également avec une plaque impériale 1200 H.D. et de centre $\alpha = 6^h\ 50^m\ 57^s$, $\delta + 25°\ 27'\ 56''$, montre comme celle du janvier 14, absence d'objet à la place mesurée sur la plaque du 1927 janvier 27.

L'objet mesuré n'est certainement pas une tache sur la plaque, mais pourrait être une étoile variable si ce n'est pas l'objet Lowell.

On the Disintegration of Short-Period Comets : Some Remarks on the Question of their Origin. By S. Vsessviatsky.

(*Communicated by Dr. A. C. D. Crommelin.*)

1. The solution of the problem of the origin of comets by the methods of celestial mechanics or of statistical investigation of the orbits meets with considerable difficulties, arising, in the first place, from the small amount of observational data available, and in the second, from the so-called conditions of visibility as established by Holetschek.

The observed distribution of the aphelia and of the eccentricities is, in fact, affected by influences resulting from seasonal conditions, from the irregular location of the points of observation on the surface of the earth, and from other causes, and it cannot be compared with a theoretical distribution. Therefore the question of applying purely astrophysical data to the solution of the problem of the origin of comets, and of applying data of the general brightness of comets, is one of very great importance.

The first steps in the statistical study of photometric data were taken by the writer of the present paper. He showed that a correlation existed between the average integral brightness of the periodic comets and the value of the semi-axis of their orbits ; * this result indicated a continual disintegration of comets under the influence of the sun or of some other factors.

The subject was further developed by Bobrovnikoff,† who, making the two assumptions that comets were born simultaneously, and that their rate of disintegration is constant, found a correlation between the dissipation function of the comets and their absolute brightness. Bobrovnikoff employed nearly the same data as those used by the present writer. The results arrived at fully confirmed the conclusions stated in the writer's paper. Nevertheless, these investigations can

* *Russian Astr. Jour.*, **2**, 68, 1924.
† *Lick Obs. Bull.*, N. 408, **14**, 28, 1929.

only be considered as the first steps taken in that new field of study, the brightness of comets. The assumption of the simultaneity of the birth of comets and of the invariability of the dissipation function can be adopted only as a first approximation in statistical investigation. Besides, it has been proved that the average brightness of parabolic comets is smaller than the average brightness of long-period comets.* This fact certainly does not stand in contradiction with the results arrived at in connection with elliptic comets, and it may be assumed that the parabolic comets form a group apart from the elliptic comets. On the other hand, the determination of the elliptic orbits with an eccentricity approaching 1 is the less precise the smaller is the observed part of the orbit; this determination depends thus on the absolute brightness of the comet, as it is obvious that a comet with a greater absolute brightness can be observed during a longer period of time than a faint one.

On this ground it may be assumed that some of the faint parabolic comets have in reality elliptic eccentricities, and this fact must to a certain extent modify the result of statistical investigations. We therefore note here as well the influence of the peculiar conditions of visibility.

As can be seen from fig. 2 in Bobrovnikoff's paper,† there can be observed a considerable dispersion of points, which has a tendency to diminish in comets with long periods of revolution. This fact may be explained, on the one hand, by the uncertainty of the determination of the period for orbits nearly parabolic. The really existing dispersion of the absolute brightness of comets is confirmed, on the other hand, by a considerable decrease in the magnitudes H_0 of parabolic comets. An examination was made of the distribution of comets according to their absolute magnitudes, only strictly parabolic or hyperbolic comets that have appeared since 1700 being taken into account. The brightness of these comets was derived from papers of the writer. ‡

The calculation for comets with a perihelion distance $q \leqslant 0{\cdot}5$ was made apart from those with $q > 0{\cdot}5$ astronomical units. Fig. 1 shows this distribution. The curves of both groups are very much alike, and their maximum is about the value $H_0 = 6^m–7^m$. The curves run nearly symmetrically, and the figures show the dispersion of absolute magnitudes to be very great. But it is quite evident that the distribution is only an apparent one, as the real number of comets of small absolute brightness ought to be much larger.

Let us estimate the relative probability of the discovery of a comet by the length of the period during which the comets may be observed owing to their visible brightness.

Studying comets with perfectly identical orbits observed in similar conditions, and leaving out of consideration all influences of the conditions of visibility as established by Holetschek, we assume in all cases the distance of the comet from the earth to be $\Delta = 1$. If H_e be the

* *Russian Astr. Jour.*, **2**, 74, 1924.
† *Loc. cit.*, p. 32.
‡ *Russian Astr. Jour.*, **2**, 68, 1924, and **5**, 40, 1928.

limiting visible magnitude that permits the comet to be discovered or observed, it is evident that

$$H_e = H_1 + 10 \log r_1.$$

H_1 denotes the absolute magnitude of the comet, and r_1 the limiting value of the radius-vector that permits the comet to be discovered or observed. The coefficient of log r was taken as = 10 according to special

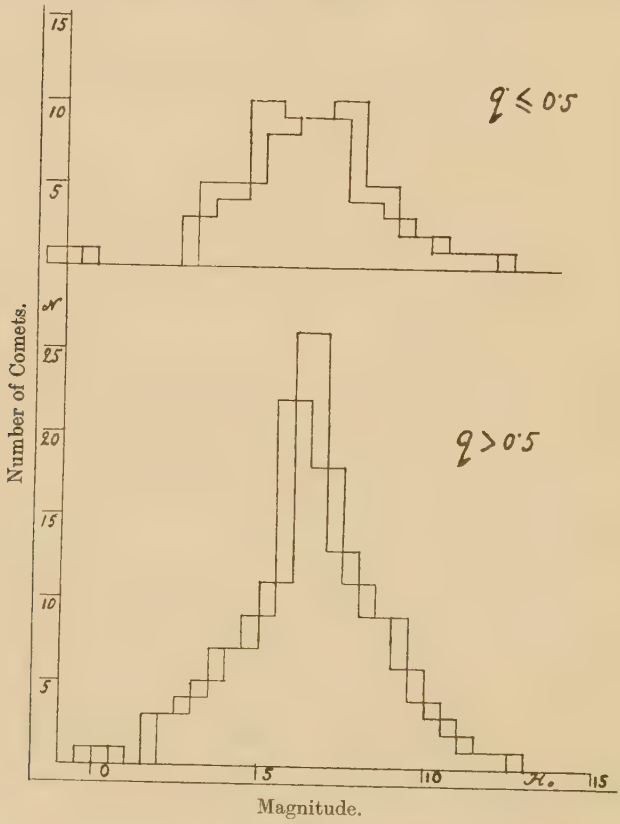


Fig. 1.

investigations. We arrive at a similar relation for a comet with absolute brightness $H_2 (H_2 > H_1)$,

$$H_e = H_2 + 10 \log r_2,$$

r_2 being in this case the limiting radius-vector. Therefore

$$H_2 - H_1 = 10(\log r_1 - \log r_2) \quad . \quad . \quad . \quad (1)$$

If $H_2 > H_1$, we obtain $r_1 > r_2$, which is obvious.

Using the equation of Euler-Lambert,

$$6\theta_1 = (r_0 + r_1 + s)^{\frac{3}{2}} - (r_0 + r_1 - s)^{\frac{3}{2}},$$

where

$$s^2 = r_0^2 + r_1^2 - 2r_0 r_1 \cos 2f,$$

we suppose in our case that $r_0 = q$, the perihelion distance of the comet.

Therefore

$$\cos 2f = \frac{2q - r_1}{r_1}$$

and

$$s^2 = q^2 + r_1^2 - 4q^2 + 2qr_1,$$

or, as can easily be seen,

$$s = \sqrt{(r_1 - q)(r_1 + 3q)}.$$

The equation of Euler-Lambert becomes then

$$6\theta_1 = [q + r_1 + \sqrt{(r_1 - q)(r_1 + 3q)}]^{\frac{3}{2}} - [q + r_1 - \sqrt{(r_1 - q)(r_1 + 3q)}]^{\frac{3}{2}}.$$

In that case the relative probability of the discovery of a comet of absolute magnitude H_2 in relation to a comet of absolute magnitude H_1, all conditions mentioned above being realised, would be

$$W = \frac{[q + r_2 + \sqrt{(r_2 - q)(r_2 + 3q)}]^{\frac{3}{2}} - [q + r_2 - \sqrt{(r_2 - q)(r_2 + 3q)}]^{\frac{3}{2}}}{[q + r_1 + \sqrt{(r_1 - q)(r_1 + 3q)}]^{\frac{3}{2}} - [q + r_1 - \sqrt{(r_1 - q)(r_1 + 3q)}]^{\frac{3}{2}}} \quad (2)$$

With the help of the formula (2) we may calculate W for different q and $H_2 - H_1$, assuming for r_1 values allowing the comets to be discovered (on the average) and determining r_2 from formula (1).

TABLE 1.

Values of W ($r_1 = 2{\cdot}0$).

$H_2 - H_1$ / q.	1·0.	2·0.	3·0.	4·0.	5·0.	6·0.
0·5	0·74	0·54	0·41	0·26	0·16	0·003
1·0	0·78	0·48	0·07	0	0	0

This table shows clearly that the real distribution of absolute magnitudes of comets must be quite different from the observed one. The probability of discovering a faint comet is very small as compared with the probability of discovering a bright one, and the fact of the discovery of several comets of a small absolute brightness (chiefly because of their close proximity to the earth) shows that the real number of such comets must be very large.

The dispersion of the absolute magnitudes must be smaller for

comets with $q = 0{\cdot}5$, and this fact seems to be confirmed by the curves of distribution cited above; comets with $q > 0{\cdot}5$ give a maximum much more pronounced. We have to repeat here that in this discussion we leave entirely out of consideration the influence of the "conditions of visibility" (the distance of the comet from the earth, etc.), which in some cases have great importance.

We see thus that the decrease of the dispersion of absolute magnitudes for comets with long periods (which, however, might be accidental, the number of such comets not being considerable) may be explained by the fact that the discovery of a comet with small absolute brightness is much less probable.

All this shows that the results of statistical investigation of the absolute brightness of comets must be accepted only with considerable limitations; therefore the study of data gathered from the observation of short-period comets is a very interesting and very important one. In this case the results will not in any way depend on any premises similar to those we are obliged to put forward in the case of a statistical investigation.

2. The question of the decrease in brightness of Encke's comet has been sufficiently elucidated in two papers of the present writer.* We must, of course, acknowledge that only the order of the rate of decrease of brightness is determined, but the gradual decrease in brightness is not doubtful.

The study of the laws of disintegration of other short-period comets is of great interest. The first deduction which can be made from a rough examination of the observations is the following: The comets lose their mass, they decompose, the rates of the process being different. The disappearance of Biela's comet, the rapid disintegration of Holmes's comet, the rapid decrease in brightness of Westphal's and other comets, show the possibility of a rapid catastrophic disruption unlike the slow disintegration of Encke's comet.

The chief difficulties encountered in the discussion of observational data are: First, a certain disparity in the observations of different returns resulting from the position of the comet in the sky, from the instruments used and from the personality of the observer; secondly, the uncertainty of the law of change in the brightness of the comet, considered as a function of the radius-vector, during a single apparition.

As has been demonstrated some time ago, the law of change in the brightness of certain comets is a more complicated function than the exponential function of the radius-vector; the exponent of r does not remain constant, but decreases with the decrease of r.† This introduces a definite error into the reduction of the observed brightness to the value of the radius-vector $r = 1$.

It is true that all comets do not change in brightness according to this more complicated law; this phenomenon may be explained by the fact that comets near perihelion are observable in a twilight sky, which diminishes their apparent brightness.

* *Russian Astr. Jour.*, **4**, 298, 1927, and **6**, 84, 1929.
† *Ibid.*, **2**, 70, 1924.

The accordance of different observations of the brightness of comets would not depend at all on errors of reduction if these observations were taken during different apparitions, at times when the comet has the same r. But in this case the accidental errors of observations would affect the result greatly.

In the case of faint short-period comets it is very difficult to determine the law of the change in the brightness with r; the observations of these comets are few and scattered.

Therefore the following method of work has been adopted: All observations were first reduced to $\Delta = 1$ (the distance from the earth); from the resulting H_Δ the quantity $10 \log r$ was subtracted; the remainder evidently corresponding to the following law of change of brightness:

$$I_\Delta = \frac{I_0}{r^4}.$$

As has been shown, this correlation very nearly corresponds to the observations of the majority of comets.*

For each apparition four different series of magnitudes H_0 were calculated, defining the brightness of the comet at that apparition.

In the first place, the largest and the smallest of the magnitudes H_0 at the given apparition were put down. The first value (H_s) evidently corresponded to the observation which noted the comet as faintest; the second (H_b) corresponded to the highest evaluation of the comet's brightness. It must be noted here that the relative precision of the magnitudes H_b and H_s is not the same. While the value H_b really seems to correspond to the general brightness of the comet, the magnitude H_s is much more affected by the influence of the instrument, of its magnifying power, and of the method of evaluation of the brightness. We must remember that many different influences always diminish the apparent brightness of the comet. Therefore we draw the conclusion: out of a series of observations, the observation showing the greatest brightness of the comet is nearest to the truth. Thus the values H_b must be considered as more precise than the magnitudes H_s.

The third value (H_m) represents the mean of all values H_0 for the given apparition. Lastly, for those apparitions for which corresponding observations were available, the values of H_0, corresponding to some definite value of the radius-vector, were taken (H_r).

A series of short-period comets observed at three or more apparitions were studied in this way with the object of investigating the secular changes in the general brightness. The observations were chiefly taken from Holetschek's *Untersuchungen*.† The observations after 1913 were collected by the writer from different sources, but chiefly from *A.N.*, *A.J.*, and *B.Z.A.N.*

3. The following Table II. shows the above-mentioned values H_b, H_m, H_s, and H_r for the observed apparitions of the short-period comets. The relative weights of all these magnitudes are certainly different. The

* *Russian Astr. Jour.*, **2**, 70, 1924.

† *Untersuchungen über die Grösse und Helligkeit*, T. IV. and V. (Wien).

greatest weight must be assigned to the results derived from the investigation of the values H_m, since they represent the mean magnitude of all observations for a given apparition. The weights of the values H_b and H_r are about equal. Generally speaking, the values H_r, as they do not depend on the law of change in the brightness of comets, might represent the best data for estimating the secular changes in the brightness of comets. But these values which necessarily must result from separate observations concerning a single definite moment are liable to contain to the full extent various errors made in the evaluations of the brightness.

TABLE II.

Faye.

Appearance.	H_b.	H_m.	H_s.	H_r(log r=0·25).
1843	4·0	4·2	..	4·4
1851	5·8	6·8		
1858	..	8·0		
1866	..	6·1		
1881	7·0	8·0	8·7	8·5
1888	8·5	9·0	9·5	9
1910	8·1	9	9·7	9·8
1925	9·5	10·1	10·3	10·0

D'Arrest.

Appearance.	H_b.	H_m.
1851	..	9·5
1857	..	9·6
1877	..	8·8
1890	..	9·5–10·1
1897	8·3	11
1910	..	10·1
1923	..	9·8

Tempel$_2$.

Appearance.	H_b.	H_m.	H_s.	H_r(log r=0·14).
1873	9·0	9·1	9·2	9·0
1878	9·1	9·4	9·9	9·4
1894	9·0	9·5	10·0	9·0
1899	9·2	9·8	10·7	9·7
1904	..	9·8	..	9·8
1915	..	..	11	11
1920	9·5	10·1	10·5	10·4
1925	10·1	11·3	12·1	10·1

Pons-Winnecke.

Appearance.	H_b.	H_m.	H_s.	H_r(log r=9·90).
1819	8·3	8·6	8·9	8·5
1858	..	9·0		
1869	..	9·6		
1886	9·5	9·9	11·1	9·5
1892	..	10·6		
1909	..	9·7		
1915	(8·4)	9·2		
1921	..	..	12·4	
1927	10·2	10·7	13	

Finlay.

Appearance.	H_b.	H_m.	H_s.	H_r(log r=0·00—0·04) after perih. pass.
1886	8·6	9·2	9·5	9·0
1893	9·3	10·1	10·9	10·0
1906	9·1	9·9	12·6	9·7
1919	10·8	10·9	11	11
1926	12·0	12·7	14·6	12

Wolf.

Appearance.	H_b.	H_m.	H_s.	H_r (log r=0·20—0·22).	H_r (0·30).
1884	5·9	7·6	9·9	6	8
1891	6·2	7·6	9·2	7·2	7·8
1898	7·3	8·0	8·5	7·9	8·5
1912	8·1	9·1	10·3	8·3	10
1918	8·0	9·9	11·0	8·1	10
1925	10·0	10·6	11·3		

Tempel$_3$-Swift.

Appearance.	H_m.	H_r(log r=0·03—0·10).
1869	13·0	13·0
1880	13·6	13·6
1891	13·8	13·8
1908	15·1	14·2

Perrine.

Appearance.	H_b.	H_m.	H_s.	H_r(log r=0·09—0·08).
1896	10·3	12·1	13·0	12·6
1909	14·0	14·5	15·9	14·6
(1922	..	14·8	..	14·8)

The object observed in 1922 is almost certainly not Perrine.

Brooks$_2$.

Appearance.	H_b.	H_m.	H_s.	H_r(log r=0·30) before p.p.	(0·30) after p.p.
1889	6·2	7·3	8·5	6·8	7·2
1896	7·0	8·0	9·8	7·0	7·8
1903	8·7	9·5	10·5	8·7	
1911	..	..	10·7		
1925	9·3	10·1	10·9	9·9	9·5

Borrelly.

Appearance.	H_b.	H_m.	H_s.	H_r(log r=0·15).
1905	8·0	9·0	10·4	8·1
1911	8·6	9·5	10·4	8·7
1918	8·9	10·2	10·9	9·3
1925	9·4	10·1	11	9·7

Kopff.

Appearance.	H_b.	H_m.	H_s.	H_r(log r=0·36).
1906	7·5	8·4	8·9	8·3
1919	7·5	8·6	9·3	
1926	..	10·8	..	10·8

Tuttle.

Appearance.	H_m.	H_r(log r=0·04—0·03).
1790	7·7	7·7
1858	7·8	7·8
1899	8·5	8·5
1912	8·6	8·3
1926	10·6	10·6

Giacobini-Zinner.

Appearance.	H_b.	H_m.	H_s.	H_r(log r=0·02).
1900	10·9	11·6	12·4	10·9
1913	10·4	11·3	12·8	11·0
1926	..	..	12·6	

A glance at Table II. shows a decided gradual decrease in the brightness of short-period comets in the course of time. The decrease of brightness is especially noticeable in Brooks's$_2$, Finlay's, Wolf's, and Faye's comets.

The results of the examination of Table II. are shown in Table III. The first column of this table shows the values of the decrease in brightness of the comet (δH) (expressed in star magnitudes) for the period of time (expressed in years) during which it was under observation.

Table III.

	δH.	Years.	ΔH.	H_0 (1900).	Φ(*ae*).	i_s. °
Encke	1·0	100	0·5±0·06	9·5	0·41	15·7
Faye	5·6	82	3·4±0·2	9·0	1·08	16·3
D'Arrest	0·5	72	0·4±0·15	9·5	0·98	16·2
$Tempel_2$	1·1	52	1·1±0·1	9·5	0·88	8·8
Pons-Winnecke	2·0	108	0·9±0·15	9·5	0·88	12·5
Finlay	3·5	40	4·4±0·2	10·0	0·93	4·5
Wolf	3·0	41	3·7±0·15	8·2	1·03	32·5
$Tempel_3$-Swift	1·3	39	1·8±0·25	14·0	0·89	11·5
Perrine	2·5	(26)	(4·9±0·6)	12·6	0·95	19·8
$Brooks_2$	2·8	31	4·5±0·3	8·5	1·08	10·3
Borrelly	1·2	20	3·0±0·3	8·7	1·02	23·2
Kopff	2·3	20	5·0±0·7	8·0	1·02	15·7
Tuttle	2·9	136	1·1±0·1	8·5	1·26	62·3
Giacobini-Zinner	0·0	26	0·0±0·2	11·0	0·91	35·2

Further is shown the mean decrease in brightness for half a century (ΔH) and H_0 for 1900. The decrease in brightness was supposed to go on uniformly, and the magnitudes of the comets at different apparitions were assumed to be linear functions of the time. All deviations from the straight line were considered as due to accidental errors. The values ΔH were determined from each series of magnitudes H_b, H_m, H_s, and H_r, with their mean error. In the final deduction of ΔH the greatest weight was given to values obtained from magnitudes H_m; the least weight was assigned to values obtained from H_s.

The absolute magnitudes of comets for 1900 were found from the magnitudes H_m; they thus define the mean absolute brightness of the comets for this time. All values relative to Encke's comet are taken from my preceding work.

The first fundamental conclusion which can be deduced from Tables II. and III. is the following: All short-period comets lose their brightness (and consequently their mass) with considerable rapidity. There is not one case of the reverse, *i.e.* of a secular increase in brightness. This conclusion is certainly very important, and it must be taken into account in every investigation about the origin of comets or of their nature.

Further, the results obtained in Table III. are examined in connection with different factors, which may be the cause of the disruption of comets.

4. In connection with the results obtained by the writer concerning the long-period comets and the analogous results obtained by Bobrovnikoff, the magnitudes H_0 (1900) were arranged according to the dissipation function $\Phi(ae) = 2 \log a + \log \sqrt{1 - e^2}$ (Bobrovnikoff, *loc. cit.*, p. 29). The values Φ(*ae*) are given in column 6 of Table III.

Fig. 2 shows the disposition of points. Encke's comet deviates strongly from the general conclusion, but the rest of the comets seem to indicate a certain connection of H_0 with $\Phi(ae)$, this connection being of the same character as that which is observed in long-period comets. However, the following shows that this connection is quite accidental.

Any correlation between the rapidity of disintegration of the comet and the function $\Phi(ae)$ is refuted by the disposition of the values ΔH in relation to $\Phi(ae)$; it seems, namely, that in the mean the greater $\Phi(ae)$ is (and therefore the less being the sun's influence), the more pronounced is the process of disintegration (fig. 3). This result contradicts absolutely the initial premises, and therefore we must admit that $\Phi(ae)$

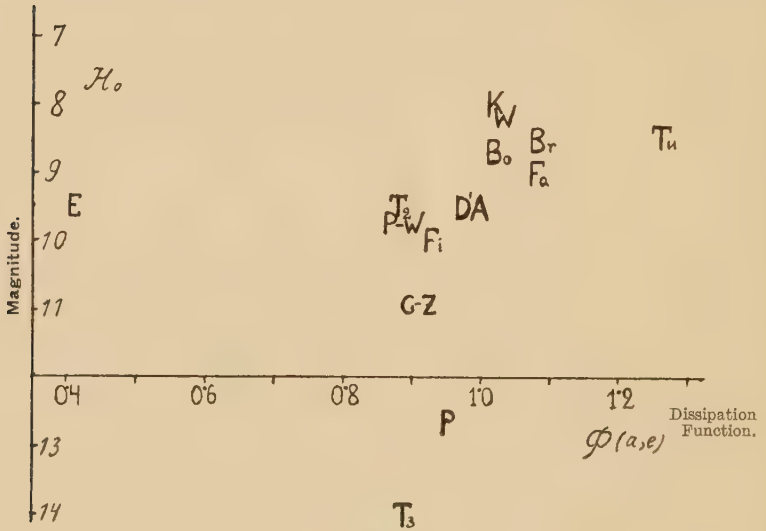


Fig. 2.

is not characteristic of the disruption of short-period comets, although for long-period comets the correlation apparently exists.

In so far as the cause of the change in the brightness of comets is sought in the sun, the position of the comet's orbit in relation to the sun's equator has to be taken into account, because near the equatorial plane the phenomena take place with a greater intensity. More particularly the existence of sun-spots shows that the maximum of the action lies in the plane of the equator. This being the case, the position of the plane of the comet's orbit relatively to the equator of the sun must play a part in the process of the disruption of the comets.

The inclination i_s of the plane of the orbit to the sun's equator was calculated by means of the formula

$$\cos i_s = \cos i \cos i\odot + \sin i \sin i\odot \cos (\Omega - \Omega\odot),$$

$i_\odot$ and $\Omega_\odot$ denoting the inclination and the longitude of the ascending

node of the sun's equator relative to the ecliptic, which values respectively are

$$7^\circ{\cdot}25 \quad \text{and} \quad 73^\circ{\cdot}67 + 0^\circ{\cdot}017(A - 1850).$$

Fig. 4 shows the distribution of the values ΔH according to i_s. The first thing which can be inferred from this figure is the possibility of separating the short-period comets into two groups according to the magnitudes ΔH. In each group there can be perceived a certain relation between ΔH and the inclination of the orbit to the sun's equator. A certain increase in the rapidity of disintegration is to be observed for small inclinations. However, this tendency is only very

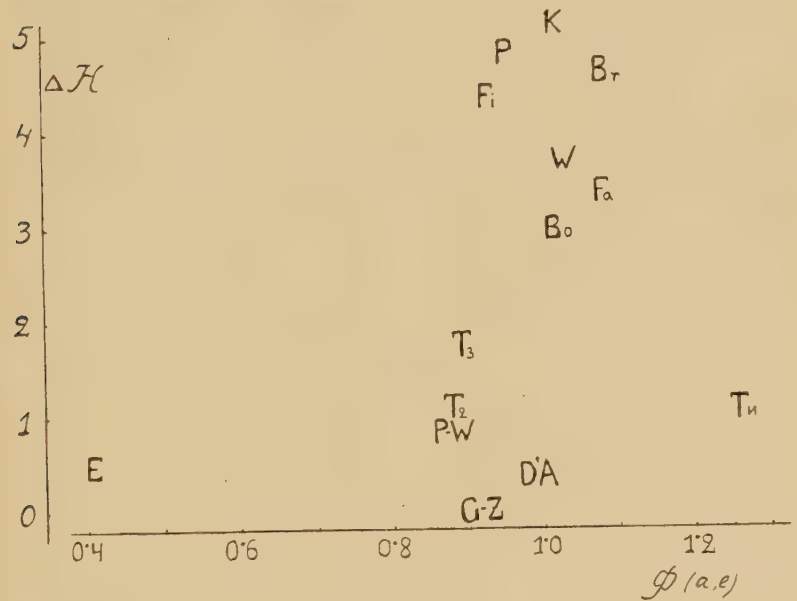


Fig. 3.

vaguely manifested, and might even be quite accidental, although it does not contradict the general ideas as to the action of the sun, and corresponds to the results obtained from comets as a whole.*

The possibility of separating the short-period comets into two groups is very interesting. Table IV. shows the members of each group.

The first group of comets is characterised by the great magnitude of ΔH, *i.e.* their disintegration is going on with a considerably greater rapidity than that of the comets of the second group. It might be inferred from Table IV. that the comets of the first group have in the mean at the same time a greater absolute brightness (Perrine's comet might be excluded, the result obtained in this case being uncertain).

* *Russian Astr. Jour.*, **2**, 74, 1924.

We must keep in mind here that the magnitudes H (1900) do not represent, in general, directly comparable elements, as they belong to different epochs in each comet's life.

TABLE IV.

Group I.	H (1900).	Group II.	H (1900).
$Brooks_2$	8·5	$Tempel_3$-Swift	14·0
Kopff	8·0	$Tempel_2$	9·5
Faye	9·0	Pons-Winnecke	9·5
Borrelly	8·7	Encke	9·5
Wolf	8·2	Giacobini-Zinner	11·0
(Perrine	12·6)	D'Arrest	9·5
Finlay	10·0		
Tuttle	8·5		

In order to be comparable with each other the absolute magnitudes H (1900) must be reduced to epochs, corresponding to the same phase of

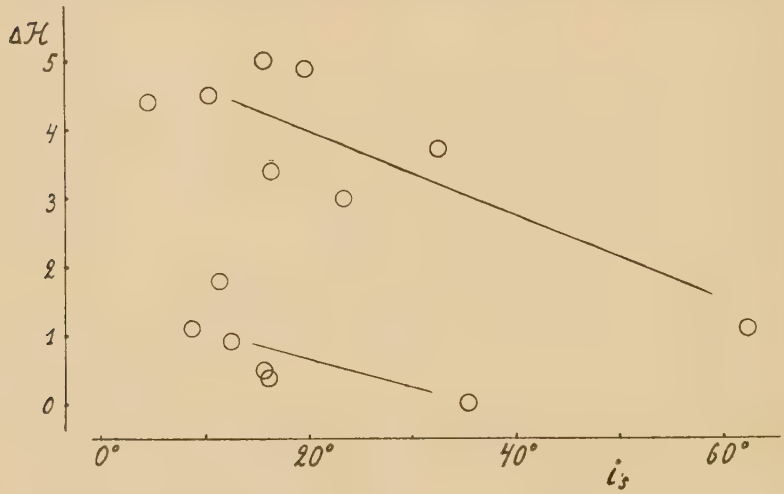


FIG. 4.

their existence. To do this with complete precision is certainly not possible, since the epochs of the birth of comets are unknown. It is nevertheless evident that the absolute magnitudes of Faye's, Finlay's, Tuttle's, and Wolf's comets have to be diminished in order to be compared with the values of other comets of the first group. The magnitudes of the comets of Group II. have to be modified accordingly, but in a smaller degree (owing to the smallness of ΔH). Fig. 5 shows the disposition of ΔH according to H (H denoting the magnitude of the comet at its first appearance). The points are very far apart, but we can see that the greater values ΔH correspond to the smaller values H; there-

fore it is possible that the rapidity of disruption of the comet is to a certain degree a function of the absolute brightness

$$\Delta H = a - bH.$$

However, long series of observations of different comets, for instance of Encke's comet, do not indicate that the decrease in brightness corresponds to the exponential law. In the case of Tuttle's comet the converse is rather to be observed, *i.e.* the growing rapidity of the decrease in brightness with the course of time. The decrease in brightness of Faye's comet also fails to follow the exponential law.

On the other hand, it is interesting to note that the rapidity of disintegration, ΔH, is in the mean greater for "young" comets, discovered comparatively recently.

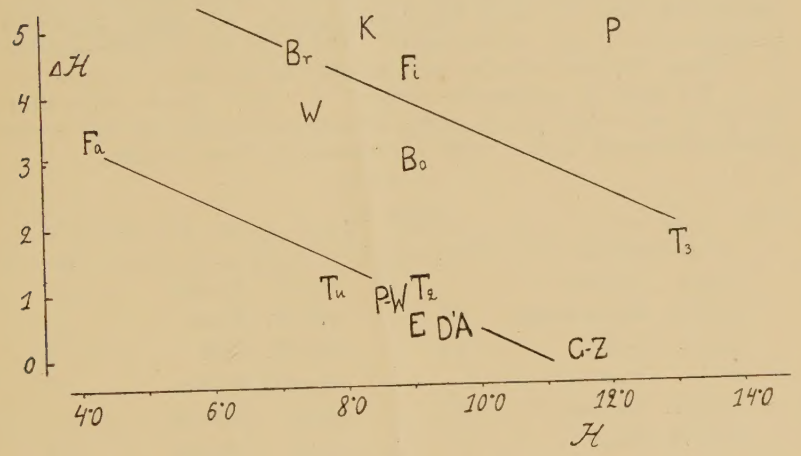


Fig. 5.

Table V. shows the comets arranged in the chronological order of their discovery.

Table V.

Before 1880.	ΔH.	After 1880.	ΔH.
Encke	0·5	Wolf	3·7
Tuttle	1·1	Finlay	4·4
Pons-Winnecke	0·9	$Tempel_3$-Swift	1·8
Faye	3·4	$Brooks_2$	4·5
D'Arrest	0·4	Giacobini-Zinner	0·0
$Tempel_2$	1·1	Perrine	4·9
		Borrelly	3·0
		Kopff	5·0

If at about 1880 we draw a line separating the "young" comets from the "old" ones, we find that the mean values ΔH for the "young"

group will be $3^{m}\cdot8$, and $1^{m}\cdot2$ for the "old" one. Only Giacobini-Zinner's comet and Faye's comet stand out. It would seem as if we saw here two groups of comets different in their physical structure, and it is of great interest to note that such a demarcation may be traced in accordance with the date of the discovery of the comet (its birth ?).

The fact that all short-period comets are disintegrating, some of them with catastrophic rapidity, indicates that we have to look for the origin of such comets in the very near past. It is impossible to admit that such comets (as, for instance, Kopff's or Borrelly's) were not discovered earlier simply because of unlucky circumstances. One hundred or two hundred years ago these comets must have been very bright. The question as to the origin of short-period comets (and perhaps of all other comets) according to this investigation must be taken up in a new way. *We must seek for the sources out of which these objects have been formed within the limits of our solar system.* The value ΔH seems to indicate the "age" of the comet. The rapid rate of disintegration of "young" comets indicates their stability to be less.

5. We know a considerable number of comets which observation proved to have short-period orbits, but which nevertheless were never observed a second time. Table VI. enumerates these comets.

TABLE VI.

	Comet.		H_0.		Comet.		H_0.
	1743 I.	Grischow	9		1884 II.	Barnard	10·5
*	1766 II.	Helfenzrieder	6·8		1886 IV.	Brooks	9·5
	1770 I.	Lexell	7·7		1889 VI.	Swift	11·5
	1783	Pigott			1890 VII.	Spitaler	9·6
	1819 IV.	Blanpain	8·5		1892 V.	Barnard	10
	1846 VI.	Peters			1894 I.	Denning	10·2
†	1858 III.	Tuttle	11		1895 II.	Swift	11
	1881 V.	Denning	9–10		1896 V.	Giacobini	9·0

In order to explain the disappearance of these comets long calculations have been undertaken for each separate case, with the purpose of elucidating the influence of the disturbing action of the major planets on the form of the orbit. Owing to the uncertainty of the initial elements of the orbit, and also owing to the nature of the question itself, it is very difficult to come to any definite conclusions in this respect. Attempts have been made to identify some of these comets with each other: thus the comet 1743 I. with the comet 1819 IV.; ‡ the comet of 1783 with the comet discovered 1881 V. by Denning; § the comets 1895 II. and 1899 V. with the celebrated comet of Lexell; || the comet 1819 IV.

* Possibly Pons-Winnecke.
† Probably identical with 1907 III. Giacobini.
‡ Clausen, *A.N.*, **10**, 345, 1833.
§ Schulhof, *B.A.*, **6**, 514, 1889; *A.A.*, **14**, 200.
|| Poor, *A.J.*, **13**, 130, 1893.

with the comet 1890 VII., and with Perrine's periodical comet; but in all these cases identification proved to be uncertain or altogether impossible.

At the present time it is no longer doubtful that the disappearance of these comets, at least of the majority of them, is due simply to the decrease in their brightness, which made it impossible to discover them in their subsequent appearances. Probably they still go on revolving round the sun, being too faint to be observed.

All the data examined prove that the secular changes in the brightness go on uniformly. The brightness of the comets is decreasing, and there is nothing to indicate that the brightness should fluctuate in some large period. Therefore one of the criteria for the identification of two comets should be the comparison of the absolute brightness of these comets. The identification is possible only when the comet discovered later has an equal or a smaller brightness.

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Erratum.

M.N., **90**, No. 6 (1930 April). Plates 11 and 12 have been accidentally transposed. Plate 12 should be Plate 11, and *vice versa*.